

Forces and Newton's Second Law

Goals and Introduction

Newton's laws of motion describe several possible effects of forces acting upon objects. In particular, Newton's second law of motion says that when there is a net force acting upon an object, the object will accelerate and its motion will change. This is often expressed as an equation,

$$F_{net} = ma \quad (\text{Eq. 1})$$

where m is the mass of the object in kg, a is the acceleration in m/s^2 , and F_{net} is the net force in N. In this lab, you will measure and quantify the relationship between the net force on an object, its mass, and the resulting acceleration.

The scenario used for our investigation in this lab is one involving two objects connected by a taut, unstretchable string, where we will ignore the mass of the string during this experiment. When two connected objects move in this kind of situation, it is important to note that the magnitudes of the velocity and acceleration of either object are always the same at any given moment in time. That is to say, if we measure the speeds of two objects connected by a taut string at a particular instant, we would find that their velocities are equal. After all, given the taut string connecting them, when one moves 1 cm the other needs to move 1 cm. If one is moving at 0.25 m/s, the other needs to move at 0.25 m/s in order to keep up. Similarly, the magnitudes of their accelerations must always be equal, or they would not remain connected with the fixed length of taut string between them. Figure 7.1 is an illustration of the apparatus for the experiment.

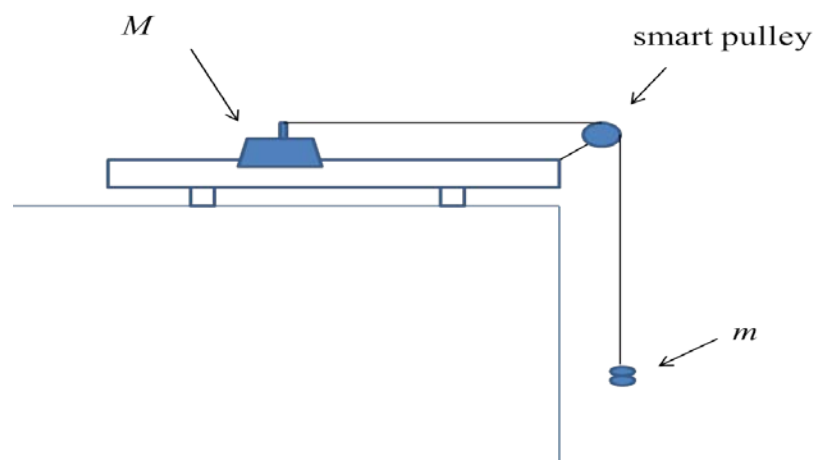


Figure 7.1

The setup consists of a glider (mass M) with an attached string extended over a pulley. The other end of the string will have a loop where metal washers (mass m) may be attached. The washers not attached to that end will be placed on the glider during the experiment. Thus, the masses M and m will change from one trial to the next as washers are moved from the glider to the free-hanging end, or vice-versa, but their combined mass ($M + m$) remains the same. The pulley is a “smart pulley” that will be connected to the computer and used to obtain a value for the acceleration of the system when set into motion.

We attempt to eliminate friction as a cause for concern in this experiment by placing the glider on the air track, as shown in Figure 7.1. Because of this, when the objects are connected and released, we expect that the washers will fall and the glider will slide to the right, since there is nothing holding it back. With friction being ignored, the force analysis becomes slightly simpler. The first step in any force analysis is to identify the objects experiencing forces and to draw a free body diagram for each object. In this experiment, the two objects with which we are concerned are the glider (and any washers on it) with total mass M , and the hanging washers, with total mass m . The forces affecting each object are identified in two free-body diagrams in Figure 7.2.

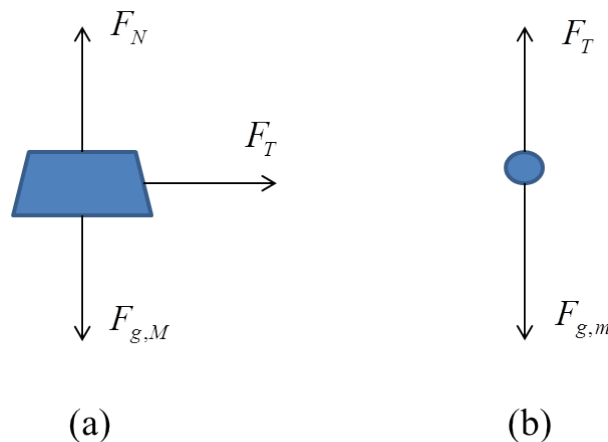


Figure 7.2

As we discuss and detail the forces affecting these objects, it may be helpful to look back at Figure 7.1 to identify the sources of these forces when possible. For example, as you have likely seen in class, strings always pull on objects. Given the way the string is attached to the glider in Figure 7.1, the tension force, F_T , would be pulling to the right on the glider. Thus, we see the tension pointing to the right in the glider’s free-body diagram (Figure 7.2a). Meanwhile, the string is connected above the washers in Figure 7.1. It is pulling up on the washers, keeping them from falling. Thus, we have shown the tension pulling up on the washers in its free-body diagram (Figure 7.2b). Recall that there can only be one value for the tension in the string at any moment.

So, the magnitude of the tension pulling on the glider is the same as the magnitude of the tension pulling on the washers.

In Figure 7.2, we also see each of the gravitational forces on each object. You will recall from class that the gravitational force on an object on Earth can be found using the equation

$$F_g = mg \quad (\text{Eq. 2})$$

where $g = 9.8 \text{ m/s}^2$ on Earth. During this experiment, you will vary the masses M and m , record their values, and later use the mass in kilograms to calculate the gravitational force on each object in newtons. The last force, appearing only in Figure 7.2a, is the normal force, F_N . This force represents the act of the air track pushing up on the glider, keeping it from falling due to the gravitational force.

With the forces identified, we must think about the expectations for the motion of these objects when they are released, and then use symbolic logic to express the effect of the forces. Without friction, there is nothing stopping the glider from accelerating to the right when the washers are connected to the string and released. The washers are going to fall and accelerate downward. This allows us to express the consequences of the net force on each object. For example, the competition of forces in the horizontal direction for the glider would require that

$$F_T = Ma, \quad (\text{Eq. 3})$$

because there is no force competing with the tension on the glider.

When we consider the free-body diagram of the washers, we expect the gravitational force to “win the tug-of-war”. One way to express the competition of the magnitudes of the forces there would be

$$F_T - F_{g,m} = -ma \quad (\text{Eq. 4})$$

Here, we have subtracted the gravitational force to indicate it points in the downward direction, and have written the right side of the equation with a minus sign to indicate that when we subtract the gravitational force from the tension, the gravitational force will “win”, causing the acceleration to be negative, or downward. By including these signs, we recognize the magnitude of the acceleration here is algebraically the same as the acceleration in Eq. 3. We can then substitute Eq.3 into Eq. 4 to solve for the acceleration.

$$a = \frac{F_{g,m}}{(M + m)} \quad (\text{Eq. 5})$$

If you don't see how to arrive at Eq. 5 through substitution, you should try the algebra using Eq. 3 and Eq. 4 to find Eq. 5. With Eq. 5, we now have the ability to predict the acceleration of the system by merely knowing the mass of each object and the gravitational force on the hanging mass!

Today, you will use the smart pulley to measure the acceleration so that you can compare it to the theoretical value, as predicted by Eq. 5, based on the masses of the glider and the hanging washers. The smart pulley is used to measure the velocity of the objects in the system. As the string causes the smart pulley to rotate, sensors inside the pulley measure the velocity of the string. By looking at the resultant graph of velocity versus time, you can calculate an acceleration by determining the slope of the velocity plot. You will then be able to calculate the percent error between the theoretical and experimental results.

- Goals:
- (1) Utilize force analysis concepts to measure and confirm Newton's second law
 - (2) Consider the analysis of two connected objects in motion, using velocity measurements to find the acceleration

Procedure

Equipment – air track, smart pulley, seven washers, glider, string, balance, computer with the DataLogger interface and LoggerPro software

- 1) Level the air track using a provided shim and/or the adjustable feet at the end of the track where the hose is attached. The track is level when, with the air turned on, the glider can be placed at any point along the track and it is able to remain nearly motionless.
- 2) Using the balance find the total mass of the 7 washers. Label this as m_{total} .
- 3) Using the balance find the mass of the glider. Label this as m_{gl} .
- 4) Check to make sure that the pulley is connected to its interface and that the interface is connected to the computer. Open LoggerPro by clicking on this link [Smart-Pulley](#). You should see a window with axes for plotting velocity vs. time.
- 5) We will now check to make sure the pulley is working. Hit the *green* button on the top-center of the screen in LoggerPro (each time you hit the *green* button, the previous plot is erased and a

new one is created). Spin the pulley slowly, using your hand, while the computer is plotting data and observe that a velocity is measured.

6) Set the glider on the air track, attach the string to the glider and drape the string over the pulley.

7) Use the procedure in the Figure 7.3 to attach 1 washer to the end of the string hanging over the pulley. Put the other 6 washers on the glider by placing them over one of its posts. Be sure to balance out the washers on either side of the glider as much as possible, as you add more during the experiments. **Record** the number of the washers on the free end and on the glider and label them as n_{w1} and n_{g1} .

NOTE: You will soon need to remove the washer and add another repeatedly, so be sure to never knot the string. Also, with the glider on the far end from the pulley, the washer should hang just below the pulley to maximize the distance between its initial position and the floor.

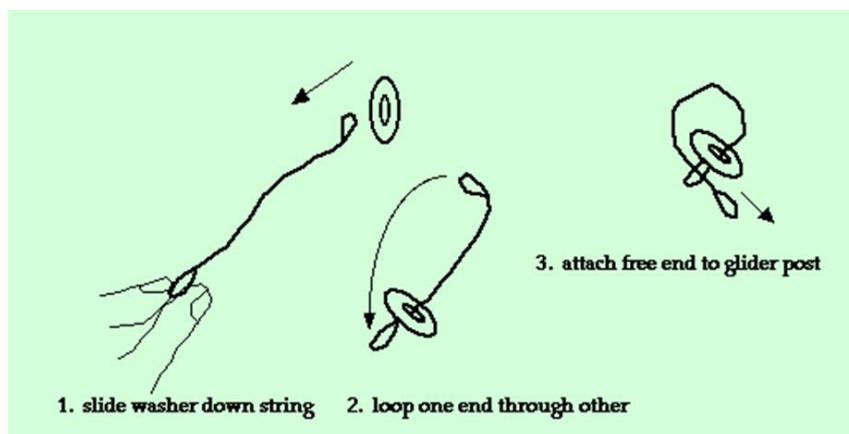


Figure 7.3

You should currently have the air supply off and the glider at rest as far from the pulley as the string will allow, with the attached string hanging over the pulley and one washer tied to its free end. When you turn on the air supply, the friction keeping the glider in place will decrease, becoming approximately zero, and the glider will begin to move as the washer falls. The pulley will measure the velocity as a function of time, and the results will be plotted on the screen. Recall that the slope on a *velocity vs. time* plot is the acceleration, or from the definition of acceleration $a = \Delta v / \Delta t$. Thus, we will now measure the acceleration of the system by finding the slope during the fall.

9) Hold the glider in place and turn on the air supply. Be ready to stop the glider so it doesn't slam into the pulley. Click on the *green* button to begin collecting velocity data. Release the

glider and stop the data collection when the washer hits the floor by clicking on the *red* button (where the *green* button was located). Then turn the air supply off.

10) Identify the straight-line portion of your graph where it looks like the acceleration (slope) was constant. Using the mouse, click and drag across the data in the straight-line portion you would like to use to find the acceleration. Then release the mouse button.

11) At the top of the screen, click on the menu “Analyze” and then select “Linear Fit”. An information box will appear that gives the slope of the line fitting your data range you highlighted. **Record** the slope (acceleration in m/s^2) and label it as a_1 . **Repeat** steps 9 through 11 until you have three values **recorded** for a_1 .

12) Close the information box and click once on the graph to clear the selected area. Reset the apparatus, placing the glider back at its starting location. Move one washer from the glider’s post to the free end of the string, so that there are now 2 tied on the free end and 5 remaining on the glider. **Record** the number of the washers on the free end and on the glider and label them as n_{w2} and n_{g2} .

13) Repeat steps 9 through 11. **Record** the acceleration in m/s^2 (the slope) and label it as a_2 . Remember to **repeat** the measurement in step 11 until you have three values for a_2 .

14) Close the information box and click once on the graph to clear the selected area. Reset the apparatus, placing the glider back at its starting location. Move one washer from the glider’s post to the free end of the string, so that there are now 3 tied on the free end and 4 remaining on the glider. **Record** the number of the washers on the free end and on the glider and label them as n_{w3} and n_{g3} .

15) Repeat steps 9 through 11. **Record** the acceleration in m/s^2 (the slope) and label it as a_3 . Remember to **repeat** the measurement in step 11 until you have three values for a_3 .

16) Close the information box and click once on the graph to clear the selected area. Reset the apparatus, placing the glider back at its starting location. Move one washer from the glider’s post to the free end of the string, so that there are now 4 tied on the free end and 3 remaining on the glider. **Record** the number of the washers on the free end and on the glider and label them as n_{w4} and n_{g4} .

17) Repeat steps 9 through 11. **Record** the acceleration in m/s^2 (the slope) and label it as a_4 . Remember to **repeat** the measurement in step 11 until you have three values for a_4 .

18) Close the information box and click once on the graph to clear the selected area. Reset the apparatus, placing the glider back at its starting location. Move one washer from the glider's post to the free end of the string, so that there are now 5 tied on the free end and 2 remaining on the glider. **Record** the number of the washers on the free end and on the glider and label them as n_{w5} and n_{g5} .

19) Repeat steps 9 through 11. **Record** the acceleration in m/s^2 (the slope) and label it as a_5 . Remember to **repeat** the measurement in step 11 until you have three values for a_5 .

20) Close the information box and click once on the graph to clear the selected area. Reset the apparatus, placing the glider back at its starting location. Move one washer from the glider's post to the free end of the string, so that there are now 6 tied on the free end and 1 remaining on the glider. **Record** the number of the washers on the free end and on the glider and label them as n_{w6} and n_{g6} .

21) Repeat steps 9 through 11. **Record** the acceleration in m/s^2 (the slope) and label it as a_6 . Remember to **repeat** the measurement in step 11 until you have three values for a_6 .

22) Close the information box and click once on the graph to clear the selected area. Reset the apparatus, placing the glider back at its starting location. Move one washer from the glider's post to the free end of the string, so that there are now 7 tied on the free end and 0 remaining on the glider. **Record** the number of the washers on the free end and on the glider and label them as n_{w7} and n_{g7} .

23) Repeat steps 9 through 11. **Record** the acceleration in m/s^2 (the slope) and label it as a_7 . Remember to **repeat** the measurement in step 11 until you have three values for a_7 .

As always, be sure to organize your data records for presentation in your lab report, using tables and labels where appropriate.

Data Analysis

You should have three acceleration measurements for each different arrangement of washers. In each case, average your three acceleration measurements. Label the averages as $a_{1\text{avg}}$, $a_{2\text{avg}}$, $a_{3\text{avg}}$, $a_{4\text{avg}}$, $a_{5\text{avg}}$, $a_{6\text{avg}}$, and $a_{7\text{avg}}$.

Question 1: Identify the trend for the acceleration as more mass is transferred to the hanging end. Is it getting bigger or smaller? Do the results fit your general expectation as to whether the acceleration should increase or decrease as mass is moved to the hanging end? Explain why.

Using the total mass of the 7 washers, m_{total} , find the mass of one washer and label it as m_w .

Recall from the introduction that the acceleration of this system could be predicted using Eq. 5 if we knew the masses, m (the hanging washers), and M (the glider with the washers on its post). You recorded the number of washers on the hanging end and the number of washers on the glider in each scenario.

Find the total mass hanging from the free end in each case, using the mass of one washer that you found, m_w . Label these results for each scenario as m_1 , m_2 , m_3 , and so on.

Find the total mass of the glider in each case, using the mass of one washer that you found, m_w , and the mass of the glider itself, m_{gl} . Label these results for each scenario as M_1 , M_2 , M_3 , and so on.

Use Eq. 2 to find the gravitational force in newtons (N) acting on the mass hanging from the free end. Label your results as F_{g1} , F_{g2} , F_{g3} , and so on.

Look back at Eq. 5. It could be rearranged to appear as $F_{g,m} = (M + m)a$. If we plot the gravitational force versus the acceleration in each case, we should ideally have a plot that is a straight line with a slope of $(M + m)$. **Create** a plot with $F_{g,m}$ on the y-axis and a_{avg} on the x-axis. This should be done in Microsoft Excel. List the accelerations in one column and the corresponding gravitational forces in the adjacent column to the right. Be sure to label your columns. Click and hold the mouse button down while dragging the cursor to highlight all the data in both columns. Then click on the “Insert” tab and then click on “Scatter” in the “Chart” section on that tab. Finally, select the scatter plot where the points are not connected:



After you create the chart, click once on one of the data points. This should cause them to be highlighted. Then click on the “Layout” tab under “Chart Tools”. Click on “Trendline” in the “Analysis” section of the tab and select “More Trendline Options”. Click the box next to “Display Equation on Chart”, and then click “OK”.

You should see an equation for the trendline on the chart. The number in front of the “x” is the slope. **Record** the slope as m_{sys} .

Now, using your results for each mass and the gravitational force in each scenario, calculate the predicted value of the acceleration in each scenario using Eq. 5. Label your results as a_{p1} , a_{p2} , a_{p3} , and so on.

HINT: Use the slope of the previous plot, $m_{\text{sys}} = (M + m)$ in Eq. 5, along with the $F_{g,m}$ values you calculated above for the seven scenarios.

You can now proceed to the error analysis section where you will determine the percent error between the measured acceleration and the predicted acceleration in each scenario.

Error Analysis

Calculate the percent error between the measured acceleration in the first scenario, a_{1avg} , and the predicted acceleration, a_{p1} .

$$\%error_1 = \frac{|a_{\text{1avg}} - a_{p1}|}{a_{p1}} \times 100\%$$

Calculate the percent error between the measured acceleration and the predicted acceleration in each of the other scenarios.

Question 2: How well did the experimental values match the predicted accelerations? What aspects of the measurement process do you feel contributed most to the differences you have calculated here?

You had found the slope of a trendline, m_{sys} . Ideally, this slope would be equal to $(M + m)$ which is equal to $m_{g1} + m_{\text{tot}}$. Find the percent error between the slope, m_{sys} , and $(M + m)$.

Questions and Conclusions

Be sure to address Questions 1 and 2 and describe what has been verified and tested by this experiment. What are the likely sources of error? Where might the physics principles investigated in this lab manifest in everyday life, or in a job setting?

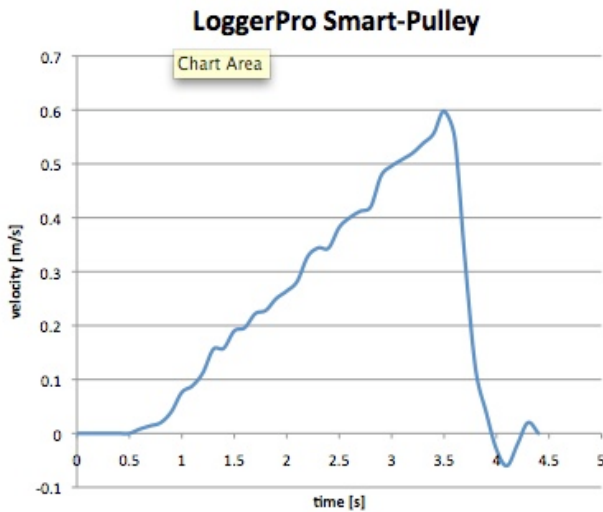
Pre-Lab Questions

Please read through all the instructions for this experiment to acquaint yourself with the experimental setup and procedures, and develop any questions you may want to discuss with your lab partner or TA before you begin. Then answer the following questions and type your answers into the Canvas quiz tool for “Forces and Newton’s Second Law,” and submit it before the start of your lab section on the day this experiment is to be run.

PL-1) The smart pulley used in this experiment measures

- the acceleration as a function of time
- the velocity as a function of time
- the force as a function of time
- the distance as a function of time

PL-2) Burt and Ernie run this experiment and get a LoggerPro plot of velocity vs. time shown in the figure. The acceleration they derive from the plot is closest to which of the following values, in m/s^2 ?



- (A) 0.00,
- (B) 0.20,
- (C) 0.35,
- (D) 0.60,
- (E) 3.0.

PL-3) Harry and Ron set up this experiment with a glider, whose mass they have measured to be 565 g, and seven washers hanging from the string. If each washer has a mass of 12 g, what is the acceleration of the system, in m/s^2 ?

PL-4) In the situation in PL-3, what is the total amount of mass (in kg) that is being accelerated?

PL-5) Which of the following conditions must be met in order for this experiment to yield valid results?

- (A) The string must stay taught during the experiment; no sagging,
- (B) The string must not stretch, so the two masses have a constant distance between them,
- (C) The string must have much less mass than the glider and/or the washers, so it has a negligible effect on the weight hanging from the pulley as more string is pulled across it,
- (D) All of A, B, and C,
- (E) None of A, B, or C – the experiment is very “robust” and will work under many conditions.