# Heat Transfer and Newton's Law of Cooling

## Goals and Introduction

When thermal energy flows out of a system or object, the temperature will decrease (if there is no phase change). Using models you have discussed in class, you could use quantities such as the *specific heat* of a substance to describe the change in temperature when an amount of thermal energy is subtracted. An object cooling in a room will eventually reach the temperature of the room, for example, and you could use the specific heat to quantify the amount of energy that must flow out of the object given its initial temperature and the room temperature. There is a question, though, as to how quickly this final temperature is achieved. After all, even our familiar thermometers require a certain amount of time after being brought into thermal contact with an object to read its temperature. While material properties certainly play a role here, it turns out that the current temperature of the object compared to what will be the final temperature is also a determining factor as to how quickly the cooling occurs at any given moment. In other words, the rate at which thermal energy is transferred, and thus the rate at which the temperature changes depends on the temperature of the object compared to its surroundings. In this lab, you will investigate Newton's Law of Cooling, which describes this phenomenon.

Newton's Law of Cooling (Eq. 1) quantifies the rate at which the temperature of an object changes ($\Delta T/\Delta t$) in terms of the current temperature of the object ($T$), the temperature of the surroundings ($T_s$), and a time constant ($\tau$) that has units of seconds.

$$\frac{\Delta T}{\Delta t} = -\frac{1}{\tau}(T - T_s) \qquad \text{(Eq. 1)}$$

Rather than use Newton's law of cooling in this form, it is often helpful to examine the behavior of the temperature of an object as a function of time while it is cooling. Eq. 1 is a differential equation with solutions for a cooling object as shown in Eq. 2, where $T_0$ is the initial temperature of the system when it begins to cool.

$$T = T_s + (T_0 - T_s)e^{-\frac{t}{\tau}} \qquad \text{(Eq. 2)}$$

Note that as time increases and goes towards infinity, the temperature of the object approaches the temperature of the surroundings, as expected. The time constant, $\tau$, depends on the material and is related to how quickly the object conducts thermal energy. Materials with a smaller time constant will cause the temperature change to be quicker, according to Eq. 2, so materials with smaller time constants are materials that conduct thermal energy more quickly.

A common method of determining the time constant is to see how long it takes for the difference between the current and surrounding temperature to fall to 37% of its initial value. The initial value of the difference between the object's temperature and the surrounding temperature would be $T_0 - T_s$. At some later time, the difference between the object's temperature and the surrounding temperature would be $T - T_s$. Eq. 2 can be rearranged to show the ratio of these two terms, as seen in Eq. 3.

$$\frac{T - T_s}{T_0 - T_s} = e^{-\frac{t}{\tau}} \qquad \text{(Eq. 3)}$$

Now, note that if we measure the temperature at the time when $t = \tau$, the right side of Eq. 3 would be $e^{-1}$ = 0.37, when rounding to the hundreths place. Thus, when $(T - T_s)/(T_0 - T_s) = 0.37$ we know the time at that moment must be equal to the time constant, $\tau$ (assuming that $t = 0$ when $T = T_0$). We can solve Eq. 3 for the temperature when the time is equal to one time constant, shown in Eq. 4.

$$T = 0.37\ (T_0 - T_s) + T_s \qquad \text{(Eq. 4)}$$

Today, you will use two different temperature probes as your objects and record their temperature as a function of time as they cool. You will then be able to analyze the data to determine the time constant for each probe and comment on which conducts heat more quickly.

*Goals*: (1) Become familiar with Newton's law of cooling and observe differences in cooling for different materials.
(2) Determine the time constant for conducting heat for two different materials.
(3) Use analytical curve-fitting to determine the time constant for each material.

**Procedure**

*Equipment* – Two styrofoam cups, thermometer, two Vernier temperature probes – one with plastic sleeve , hot plate, tea kettle, computer with the DataLogger interface and LoggerPro software

1) Put 3 cups of water in the tea kettle, Turn the burner on "high", and place the kettle on the burner so that the water will eventually boil.

2) Connect the temperature probes to DIG/SONIC 1 and DIG/SONIC 2 of the DataLogger interface. Plug the plastic-sleeved probe into DIG/SONIC 2.

3) Open LoggerPro by clicking on the link for the Temperature Probes on the lab web page.

4) On the screen, you should see an empty graph window for Temperature vs. Time. Double-click on the graph and make sure that the range of the time axis is set from 0 to 500 s.

5) Place the cups together, on inside the other, so that you effectively have one cup. This will help insulate the water during the early part of the experiment.

6) The probes must be calibrated. Begin this as soon as the water is beginning to boil. In LoggerPro, click on the "Experiment" drop-down menu and select "Calibrate" and then select one of the probes.

7) A window will pop up. Select "One Point" calibration, check the boxes next to both "CH 1" and "CH 2", and then click on "Calibrate Now".

8) When the voltage reading for "Reading 1" appears to be stable, look at your thermometer (which is measuring room temperature currently) and type in the temperature in °C. Then click "Keep" and then click "OK".

9) **Record** the room temperature from the mercury thermometer and label it as $T_s$.

10) Once the water is boiling, fill the cup with boiling water, click on the *green* button to begin data collection in LoggerPro, and put both probes in the cup.

11) Watch the plots of temperature being drawn on the screen. After about 75-100 s, you should notice that the temperatures have leveled off and maybe even begin to decrease. As soon as you notice this behavior, remove the probes from the cup. While holding the probes above the table, allow the temperature plot to be created for the remaining time until the total time of 500 s is reached. Be sure that the probes do not touch each other or anything else while you are recording data. DO NOT erase the graph or close LoggerPro until you have finished the Procedure section.

12) **Print** two copies of your graph, one for each lab partner. Alternatively, you might explore options to **Save** your plot as an image and email it to you and your lab partner so that you each have a copy for your lab report.

13) Click on the drop-down menu "Analyze" and select "Examine" so that it appears to be checked on the menu. Then, when you float the cursor over either of the lines on the graph, it should show you the time and temperature at that point.

14) Float the cursor as closely as possible over the point where the plot for the uncovered probe is beginning to cool down. **Record** the time at this point when the cool down begins and label it as $t_{0u}$ . **Record** the temperature of the uncovered probe at this time and label it as $T_{0u}$ .

15) Float the cursor as closely as possible over the point where the plot for the covered probe is beginning to cool down. **Record** the time at this point when the cool down begins and label it as $t_{0c}$ . **Record** the temperature of the covered probe at this time and label it as $T_{0c}$ . Note that it is possible the values for both the uncovered and covered probes will be very similar.

16) We now must do a quick calculation using Eq. 4. Using the room temperature, $T_s$, and the initial temperature $T_0$ in each case, **compute** the temperature $T$ for each probe when one time constant has passed. Label your results as $T_u$ and $T_c$. Detail this calculation showing your work so that you can present this in your lab report.

17) Go to the plot for the uncovered probe and float the cursor over the location when the temperature, $T_u$ is reached. **Record** the time and label it as $t_{1u}$ .

18) Go to the plot for the covered probe and float the cursor over the location when the temperature, $T_c$ is reached. **Record** the time and label it as $t_{1c}$ .

19) Click and drag on the plot so that the last 200 s or so of the cool down is highlighted. Then click on the drop down menu "Analyze" and select "Curve Fit". A dialog box will open. Select "latest|temperature1."

20) Select "Natural Exponent" for the general equation. Then click on "Try Fit". The equation and its coefficients are shown on the right side of the window. **Record** the equation with its coefficients and note that it is for the uncovered probe. Then click "OK."

21) Make sure that the same time range is still highlighted, or highlight the last 200 s or so of the cool down again, and click on the drop down menu "Analyze" and select "Curve Fit". A dialog box will open again. This time select "latest|temperature2."

22) Select "Natural Exponent" for the general equation. Then click on "Try Fit". **Record** the equation with its coefficients and note that it is for the covered probe. Then click "OK."

As always, be sure to organize your data records for presentation in your lab report, using tables and labels where appropriate.

## Data Analysis

During the procedure, you used Eq. 4 to determine the temperature of each probe when one time constant had passed. You then recorded the time when this temperature was reached by each probe, $t_{1u}$ and $t_{1c}$. This time measurement was not referenced to when the cool-down began, however.

Find the time constant, $\tau$, for each probe by subtracting the beginning cool-down time, $t_{0u}$ or $t_{0c}$, from the recorded time when one time constant had passed, $t_{1u}$ and $t_{1c}$. Label your results as $\tau_{u,A}$ and $\tau_{c,A}$.

**Question 1:** Consider your graphs and the time constants you measured for each probe. Which probe cooled faster, the covered or uncovered probe? Does a larger time constant indicate an object that is cooling more quickly, or more slowly than one with a smaller time constant? Explain.

Now, during the procedure, you had recorded a best-fit equation for each probe's cooling profile. Compare that equation to Eq. 2. The value that appears in front of the "*t*" in each equation should be equal to $-1/\tau$ for that probe. Compute the time constant, $\tau$, using the information in the equations you recorded and label your results as $\tau_{u,B}$ and $\tau_{c,B}$.

**Question 2:** Look up and define the following three methods of energy transfer: *conduction*, *convection*, and *radiation*. Which of these three methods is most relevant in explaining how the probe cools during this experiment? Explain.

## Error Analysis

The percent difference between two quantities, *X* and *Y*, would be given by the equation shown below.

$$\%diff = \left| \frac{X - Y}{(X + Y)/2} \right| \times 100\%$$

Compute the percent difference for your two measurements of the time constant for the uncovered probe.

Compute the percent difference for your two measurements of the time constant for the covered probe.

## Questions and Conclusions

Be sure to address Questions 1 and 2 and describe what has been verified and tested by this experiment. What are the likely sources of error? Where might the physics principles investigated in this lab manifest in everyday life, or in a job setting?

## Pre-Lab Questions

Please read through all the instructions for this experiment to acquaint yourself with the experimental setup and procedures, and develop any questions you may want to discuss with your lab partner or TA before you begin. Then answer the following questions and type your answers into the Canvas quiz tool for “Heat Transfer and Newton’s Law of Cooling,” and submit it before the start of your lab section on the day this experiment is to be run.

PL-1) The time constant, $\tau$, has units of
(A) degrees Celsius.
(B) newtons.
(C) seconds.
(D) meters/second.
(E) joules.

PL-2) Juan and Maria are performing the Heat Transfer and Newton’s Law of Cooling experiment. After Juan and Maria place the probes in the boiled water, they should remove the probes when
(A) they reach a temperature of 100 °C.
(B) they have been cooling for at least 50 s.
(C) the entire 500 s have passed.
(D) their temperature has stopped increasing and appears to have leveled off.
(E) they stop measuring temperature.

PL-3) When looking at their graph, Juan and Maria note that their uncovered probe begins to cool after reaching a temperature of 85 °C. If the room temperature was 23 °C, what should the probe temperature be after one time constant has passed?

PL-4) Juan and Maria note that the uncovered probe began to cool when $t$ = 156 s on their graph. After calculating the temperature when one time constant has passed, they find that this temperature is reached when $t$ = 327 s. What is the time constant as measured by this probe?

PL-5) If Juan and Maria were to find the difference in temperature between the probe temperature and the surroundings after an amount of time equal to *two* time constants had passed, it would be what? *[Consider the discussion of Eq. 3 and the derivation of Eq. 4 in the Goals and Introduction section, for aid in answering this question]*

(A) equal to zero.

(B) about 14% of the difference between the initial temperature and the surrounding temperature.

(C) about 37% of the difference between the initial temperature and the surrounding temperature.

(D) about 74% of the difference between the initial temperature and the surrounding temperature.

(E) about 50% of the difference between the initial temperature and the surrounding temperature.