Rotational Dynamics

Goals and Introduction

In translational dynamics, we use the quantities displacement, velocity, acceleration, mass and force to model the motion of objects. In that model, a net force acting on an object with some amount of mass will cause that object to accelerate and change its motion. The position and velocity of the object can then be predicted later on in time based on its initial position and velocity, and this acceleration (as long as it is applied). Similarly, an object that is rotating is in motion and can have its rotational motion changed. However, the quantities of displacement, velocity and acceleration are difficult to apply in this scenario where the object is not moving in-line, or “translating.”

At a fundamental level, it is true that forces cause an object’s motion to change. However, it is possible that an applied force can cause an object to rotate rather than translate. The ability of a force to cause rotation is called a *torque* (τ) and the change in rotational motion that results from an applied torque depends on the mass of the object and its size. After all, if you consider applying a force to the edge of a door in order to cause it to rotate about its hinges, it is much easier to rotate a lighter door than it is a heavier door of the same size and shape (imagine a screen door and the door to a bank vault). Likewise, two different doors could have the same mass and height, but one could be made of a less dense material, causing it to be wider. That “less dense door” would have a different change in rotational motion from an applied torque than the other door. The physical quantity that accounts for the differences between all of these doors, and other objects, is called the *moment of inertia*, I. In this lab, you will work to develop an understanding of how to model rotational motion and quantify several aspects in order to measure the moment of inertia of an object and compare to its theoretical value.

Imagine looking down at an object that is rotating, such as a wheel or disk, as shown in Figure 13.1. The angular position of the object can be tracked by using a point on the outer edge of the object and mapping its location as the object spins. If the object is rotating, the point is seen to move from one position to another and it can be said that the object has undergone an *angular displacement*, $\Delta\theta$.

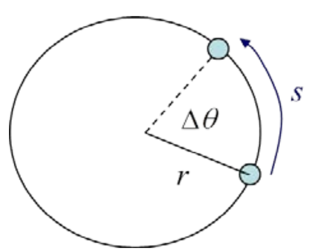

Figure 13.1

For the case of an object with a radius, r, such as that shown in the figure, the distance traveled by the point, s, is an arc length and is related to the angular displacement by

$$s = r\Delta\theta \qquad \text{(Eq. 1)}$$

where $\Delta\theta$ is measured in radians. For the purposes of this lab, we will measure all angular displacements as positive.

Given that it takes a certain amount of time for this angular displacement to occur, an average *angular speed*, ω_{avg}, can be found in a similar fashion to how we found an average translational velocity, v_{avg}, earlier in the semester. As shown in Equation 2, the average angular speed would be the angular displacement divided by the time over which the change occurred, and would have units of radians per second (rad/s).

$$\omega_{avg} = \frac{\Delta\theta}{\Delta t} \qquad \text{(Eq. 2)}$$

Again, the rotational motion of the object can be connected to the translational motion of the point on the object by observing that as the object goes through an angular displacement in a certain amount of time, the point travels a distance, s. Thus, the point must be moving with some average speed, v. This average speed would depend on the distance between the point and the center of rotation, as this would affect the distance traveled. Yet, all points on the object must have the same angular velocity at any moment! This truth is summarized in Equation 3, where the speed of any point on the object can be found by knowing the instantaneous angular speed and the distance of the point from the center of rotation.

$$v = r\omega \qquad \text{(Eq. 3)}$$

Now, the object's rotational motion could be changing, and just as we did in translational mechanics, we can identify this is occurring because the speed may be changing. In other words, the rotational motion is changing if there is some *angular acceleration*, α (we will be using models involving constant acceleration). This is related to the point on the object undergoing a change in speed or experiencing a tangential acceleration, a_t. By definition, the angular acceleration of an object is equal to the change in angular speed over the time in which that change occurs (Eq. 4). This is related to the tangential acceleration of any point on the object by the distance of that point from the center of rotation (Eq. 5), very similar to how Equations 1 and 3 related the translational motion of the point to the rotational motion of the object.

$$\alpha = \frac{\Delta\omega}{\Delta t} \qquad \text{(Eq. 4)}$$

$$a_t = r\alpha \qquad \text{(Eq. 5)}$$

Finally, we can relate the applied torque on an object to a resulting angular acceleration, just as we related an applied force to a resulting acceleration in Newton's second law ($F = ma$) earlier in the semester. Torques cause the rotational motion to change, which is observed by measuring an angular acceleration. As shown in Equation 6, the resulting angular acceleration depends on the moment of inertia of the object, just as the resulting acceleration from an applied force depended on the mass before (thus, the moment of inertia behaves rather like mass in that it resists changes in motion).

$$\tau = I\alpha \qquad \text{(Eq. 6)}$$

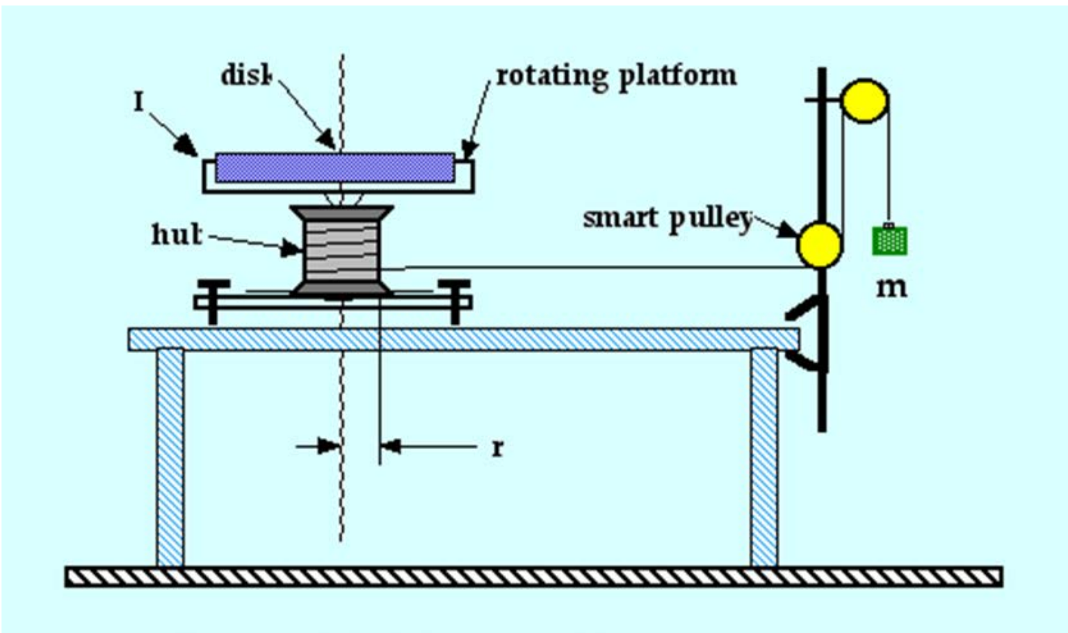

Figure 13.2

The apparatus used in this lab is shown in Figure 13.2. The rotating components of the experiment consist of a hub with radius r, attached to a platform upon which a disk may be placed. A cord that is wrapped around the hub is attached to a mass holder. When mass is placed on the end and is allowed to fall, the gravitational force on the mass will create a tension in the cord which will cause the hub to rotate by applying a torque. The amount of torque applied to the hub will be

$$\tau = mgr \qquad \text{(Eq.7)}$$

where *mg* is the gravitational force, or weight, of the mass. The mass should be in kg, and the radius in m. This torque is illustrated by the top view shown in Figure 13.3.

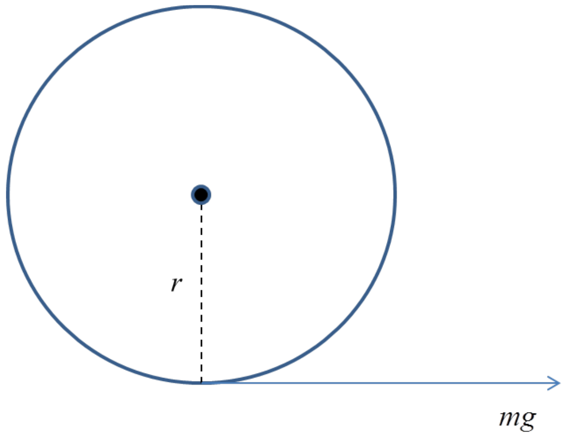

Figure 13.3

A smart pulley will be used to measure the acceleration of the mass. Because this is related to the movement of the string and the string is wrapped around the exterior of the hub, the smart pulley is also measuring the tangential acceleration, a_t, of a point on the edge of the hub, a distance r from its center of rotation. By measuring tangential acceleration, it is possible to find the angular acceleration, α, of the hub and any components attached to it (the platform and the disk).

It is important to note that when components like the platform and disk are rotating with the hub, we are able to sum their individual moments of inertia to find the total moment of inertia. For example, the formula for the moment of inertia of a disk is

$$I_{disk} = \frac{1}{2}MR^2 \qquad \text{(Eq. 8)}$$

where M is the mass of the disk and R is the radius of the disk. If we know the moment of inertia for the hub/platform combo, the total moment of inertia would be given by Equation 9.

$$I_{total} = I_{hub/plat} + I_{disk} \qquad \text{(Eq. 9)}$$

Since the shape of the hub/platform is quite irregular, unlike the disk, there is not an easy way to determine the hub/platform’s moment of inertia mathematically. Therefore, you will be experimentally determining the hub/platform moment of inertia, and using that result in hour calculations.

In this lab, you will use the smart pulley to measure tangential acceleration, a_t, and then calculate the angular acceleration, α, of the hub/platform system. You will then predict the amount of added mass necessary to achieve the same angular acceleration with the disk attached, and test your predictions.

Goals: (1) Develop a better understanding of rotational kinematics and the relationships between torque, angular acceleration, and tangential acceleration.

(2) Predict the necessary applied torque and force to cause a particular change in motion and test your prediction

Procedure

Equipment – hub and platform system with pulleys including a smart pulley, computer with DataLogger interface and LoggerPro software, disk, a set of removable masses, two mass holders, meter stick or caliper

1) Check to make sure that the pulley is connected to its interface and that the interface is connected to the computer. Open LoggerPro by clicking on the link on the course website. You should see a window with axes for plotting velocity vs. time. Click on the time in the lower right corner of the plot and change the final value to 5 s.

2) We will now check to make sure the pulley is working. Hit the *green* button on the top-center of the screen in LoggerPro (each time you hit the *green* button, the previous plot is erased and a new one is created). Spin the pulley slowly, using your hand, while the computer is plotting data and observe that a velocity is measured.

Because we would like to measure the moment of inertia of the hub/platform combo, we would like to first eliminate the effects of friction between the hub and the axle about which it rotates. There is only so much we can physically do to reduce this effect, so we will instead try to compensate for the issue by determining the amount of mass necessary to counter the effects of friction on the axle. This will be evident if the attached mass causes the hub to rotate at a constant angular velocity (which means the pulley records a constant velocity), as it falls.

3) **Measure and record** the mass of the lighter of the two mass holders. Then, hang it from the free end of the cord as shown in Figure 13.2. Label this as m_{light}. Attach the other end of the string to the small peg on the side of the hub and wind the platform to pull the string up so that the mass holder is at its highest point, near the pulley. Then hold the platform in place.

4) By trial and error, determine the amount of additional mass necessary to cause the hub to rotate with a constant velocity when the holder and the additional mass fall. To measure this, attach mass to the holder while it is hanging. Hit the *green* button on the top-center of the screen

in LoggerPro and then release the mass/holder, allowing it to fall to the floor. **Record** the total mass necessary to compensate for friction – the mass of the holder and the additional mass added. Label this as m_{f1}.

HINT: You will probably need several attempts to find the right mass for the previous step. The result will probably lie between 5 and 50 g. It is important to get this right because any additional mass beyond this amount is what will really be causing acceleration, as measured by the pulley. This amount of mass must always be subtracted from the totals we measure later, to account for the effects of friction.

5) We will now make measurements to find the moment of inertia of the hub/platform. **Choose and record** an additional amount of mass to be the driving mass for the hub, and label it as m_{hub1}. Add your driving mass to m_{f1} and hang this total mass from the free end of the cord.

HINT: If you need to switch to the other mass holder at any time, be sure to measure its mass and add mass to it in order to achieve your friction-compensating mass, m_{f1}. That amount of mass should always be present before adding any driving mass.

6) Hit the *green* button on the top-center of the screen in LoggerPro and then release the mass/holder, allowing it to fall to the floor. Then hit the *red* button to stop the data collection.

7) Identify the straight-line portion of your graph where it looks like the acceleration (slope) was constant. If the straight-line portion occurs over less than 1 s, choose a lesser driving mass and repeat the last step. Using the mouse, click and drag across the data in the straight-line portion you would like to use to find the acceleration. Then release the mouse button.

8) At the top of the screen, click on the menu "Analyze" and then select "Linear Fit". An information box will appear that gives the slope of the line fitting your data range you highlighted. **Record** the slope (acceleration in m/s^2) and label it as a_{hub1}. **Repeat** steps 6-8 until you have ten values **recorded** for a_{hub1}. This is tangential acceleration.

HINT: Note that you may need to reattach the string to the peg on the hub in order to rewind the string after each trial. Alternatively, when allowing the string to unwind from the hub, stop the hub from rotating just before the end of the string has been reached, so that the string doesn't fall off of the peg. This will allow you to more easily rewind the string around the hub for each of your remaining trials. DO NOT tie the string to the hub, as this can cause the string to slip into the axle, and become permanently stuck.

9) **Calculate** the mean of your ten accelerations. Then, use this mean to calculate an angular acceleration of the hub/platform (see Figure 13.3 and the Goals and Introduction for help thinking through this).

10) **Measure and record** the diameter of the hub using the caliper (or ruler if no caliper is present). Then, **calculate** the radius of the hub, labeling it as r.

11) Using your driving mass and Equation 7, **calculate** the torque that is being applied to the hub (we have already compensated for friction).

12) Now, using your torque, your angular acceleration, and Equation 6, **calculate** the moment of inertia of the hub/platform.

13) Now, we will add a disk to the platform. **Measure and record** the mass of the disk (M) and the diameter of the disk. Then, **calculate** the radius of the disk, labeling it as R.

14) **Calculate** the predicted moment of inertia of the disk, I_{disk}, using Equation 8.

15) **Calculate** the predicted total moment of inertia when the disk is added to the hub/platform, using your calculated moment of inertia for the hub/platform, and your predicted value of the disk's moment of inertia. Label this as $I_{predict}$.

16) Place the disk on the platform. Using an analysis similar to one you just applied, determine the necessary driving mass to cause the hub/platform/disk system to rotate with an angular acceleration equal to your mean value found in step 9. **Record** the driving mass and label it as m_{hub2}.

17) **Measure and record** the mass of the heavier of the two mass holders. Then, hang it from the free end of the cord as shown in Figure 13.2. Label this as m_{heavy}.

18) Using the heavier mass holder, by trial and error, determine the amount of additional mass necessary to cause the hub to rotate with a constant velocity when the holder and the additional mass fall. **Record** the total mass (the mass of the holder and the additional mass added) necessary to compensate for friction now that we have added the disk to the platform. Label this as m_{f2}.

19) Add your driving mass, m_{hub2}, to m_{f2} and hang this total mass from the free end of the cord.

20) Hit the *green* button on the top-center of the screen in LoggerPro and then release the mass/holder, allowing it to fall to the floor. Then hit the *red* button to stop the data collection.

21) Identify the straight-line portion of your graph where it looks like the acceleration (slope) was constant. Using the mouse, click and drag across the data in the straight-line portion you would like to use to find the acceleration. Then release the mouse button.

22) At the top of the screen, click on the menu “Analyze” and then select “Linear Fit”. An information box will appear that gives the slope of the line fitting your data range you highlighted. If your acceleration (the slope) is not somewhat similar to the mean of a_{hub1}, check your calculations in step 16 and consult your TA. **Record** the slope (acceleration in m/s^2) and label it as a_{hub2}. **Repeat** steps 20-22 until you have ten values **recorded** for a_{hub2}. Again, this is tangential acceleration.

Data Analysis

Be sure to show all of your calculations from the Procedure section either there or here in the Data Analysis section – your choice for presentation.

Calculate the mean value of the ten accelerations measured for the hub/platform, before the disk was added.

Calculate the mean value of the ten accelerations measured for the hub/platform/disk, after the disk was added.

Using a process similar to that which you performed in the procedure, use the mean value of the acceleration for the hub/platform/disk to find the total moment of inertia of the hub/platform/disk. Label your result as I_{exp}.

Error Analysis

Calculate the percent error between the mean value of the hub/platform acceleration, a_{hub1}, and the mean value of the hub/platform/disk acceleration, a_{hub2}.

Calculate the percent error between your predicted total moment of inertia, $I_{predict}$ (from Eq. 9), and the experimental moment of inertia, I_{exp}.

Question 1: Should these percent errors be similar? Explain your reasoning.

Questions and Conclusions

Be sure to address Question 1 and describe what has been verified and tested by this experiment. What are the likely sources of error? Where might the physics principles investigated in this lab manifest in everyday life, or in a job setting?

Pre-Lab Questions

Please read through all the instructions for this experiment to acquaint yourself with the experimental setup and procedures, and develop any questions you may want to discuss with your lab partner or TA before you begin. Then answer the following questions and type your answers into the Canvas quiz tool for "Rotational Dynamics," and submit it before the start of your lab section on the day this experiment is to be run.

Adam and Jamie are doing the Rotational Dynamics lab "on a grand scale." They weld a 55-gallon oil drum (diameter 0.59 m, height 0.91 m) onto a steel bracket designed to hold a large concrete disk. Together, this hub (the drum) and platform can rotate around a vertical axis on a low-friction bearing as shown in Figure 13.2 (see the lab Introduction). They wrap a heavy steel cable around the circumference of the drum and lead it through two pulleys, as shown, so that different masses can be attached. The weight of these masses provides the force tangent to the surface of the drum - a torque - that in turn provides the angular acceleration of the hub, platform, and (when it is installed) the concrete disk.

First, Adam and Jamie add just enough mass to the end of the wire to overcome the friction of the system, so the weight drops at a constant velocity and the hub and platform (the disk has not been installed yet) rotate at a constant angular velocity. Then, they add more weight to the end of the wire (Adam hangs from it, Adam has a mass of 78 kg), and measure his downward acceleration by analyzing their video frame-by-frame. Following Steps 5-12 of the Procedure, they calculate the moment of inertia of the hub and platform (without the concrete slab) to be $I_{\text{hub1}} = 550\ \text{kg·m}^2$.

PL-1) Predict the moment of inertia, in kg·m^2, of the concrete disk (Adam rescued it from the garage of a celebrity mansion where it was used as a turntable for turning cars around; apparently the owner did not like backing out of the garage!). The disk is 3.6 m in diameter and 0.23 m thick, and has a mass of 5619 kg.

PL-2) Adam and Jamie use a crane to lower the concrete disk gently onto the hub/platform. They hang an old Volkswagen Beetle (mass 826 kg) from the wire and videotape it while it falls. They do a frame-by-frame analysis of their video to find that the car accelerated downward at a rate of 0.073 m/s^2. Assuming the steel cable was taught and did not stretch, calculate the angular acceleration of the hub/platform/disk, in radians per second-squared (rad/s^2). Recall that the diameter of the drum is 0.59 m. *[Hint: see Steps 9-12 of the Procedure and consider Figure 13.3 in the lab]*.

PL-3) Calculate the amount of torque, in N·m, that the cable exerts on the hub/platform/disk in the previous example. Recall the mass of the Beetle is 826 kg and the diameter of the drum is 0.59 m. *[Hint: see Steps 9-12 of the Procedure and consider Figure 13.3 in the lab]*.

PL-4) Using your torque and angular acceleration found in the previous two questions, calculate the total moment of inertia of the entire hub/platform/disk assembly in $kg{\cdot}m^2$. *[Hint: see Steps 9-12 of the Procedure]*.

PL-5) Calculate the moment of inertia, in $kg{\cdot}m^2$, for the disk alone. Recall that the moment of inertia they measured for the hub/platform was 550 $kg{\cdot}m^2$.