Statistical Analysis of Data

Goals and Introduction

In this experiment, we will work to calculate, understand, and evaluate statistical measures of a set of data. In most cases, when taking data, there is going to be some variance in the measurement process, even when repeating the process again and again. For example, imagine you are trying to measure the length of time it takes for an object to fall from some height to the floor (x). You drop the object three times, and each time you measure the fall time with a stopwatch, obtaining 1 ms, 10 ms, and 19 ms for the three trials (1 ms is one millisecond). In statistical terms, your *data set* consists of three numbers, {1, 10, 19} ms. If you repeat the experiment using a better timing device, you might obtain a second data set such as {9, 10, 11} ms. Both of the data sets have the same average, or *arithmetic mean* ($\bar{x}$), but there are some definite differences which speak to our level of certainty of the measurement process in each case. In which mean are you most confident? If a fourth measurement were made when each data set was taken, within what range from the mean could you confidently say the measurement would fall? Our uncertainty in the measurement and our confidence in the value of the mean can actually be quantified using statistics – in particular, the *standard deviation* (σ) and the *standard deviation of the mean* (σ_m), respectively. This kind of information about a measurement, or quantity, is very important when communicating results not just in any of the sciences, but in economics, business, and even sports!

Students sometimes become frustrated when they are asked to make several measurements of the same quantity, but one only needs to look at the two datasets above to see why this is important. There is a big difference between the sets, and we can discuss our results with a different level of confidence in one case versus the other. We would like to be certain that we have the correct mean value. There are some similarities between the datasets though. They are each comprised of three measurements, all of the measurements are between 0 and 20 ($0 < x < 20$), and the arithmetic mean of each dataset is the same ($\bar{x} = 10$). The arithmetic mean, or *mean*, of a dataset is the same as the mathematical average of the data, as seen in Eq. 1.

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N} \qquad \text{(Eq. 1)}$$

In Eq. 1, N is the total number of measurements and each x_i represents one of the measurements. The Greek letter *sigma* in the numerator of Eq. 1 tells us to add up all of the measurements in the dataset. For example, for the first dataset above, the numerator of Eq. 1 would be equal to

1 + 10 + 19 = 30. We then divide by the number of measurements (3 for either above dataset). This gets you the *mean* of the dataset. Check now to verify that the mean of each of the above datasets is indeed equal to 10.

This is where the similarities between the datasets end, however. We can see that the individual measurements are very near the mean in one case, and predominantly far from the mean in the other. If we were to make a fourth measurement using each method, we would expect the result to be closer to the mean using the second method, rather than the first. This speaks to an uncertainty in the next measurement that might be made in each case. Another way of describing this is that there must be some "average measurement error" given the conditions of the experiment. A numerical value that can be calculated, which describes this uncertainty in the next measurement and how far it will fall from the mean, is the *standard deviation* (σ). Eq. 2 illustrates how to calculate the standard deviation.

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \bar{x})^2}{N-1}} \qquad \text{(Eq. 2)}$$

This may seem like a very complicated formula, and it may not be clear how this describes the uncertainty in the next measurement, so let's consider its various parts. First the numerator contains a sum of a squared quantity, $x_i - \bar{x}$, called the "deviation from the mean". This is the difference between each measurement and the mean of the dataset. You might say, "Hey, that's what we want. If it's big, the data is all over the place, and if it's small, the data is clustered tightly around the mean." Remember though that we need to calculate this difference for each data point! If we just added up those results, we would ALWAYS get a final result of 0. Don't just trust me. Try it with each of the data sets above. If you just sum all of the deviations from the mean in each case, you get 0. So that alone isn't helpful. This is because some of the deviations are negative and some are positive, since some of the measurements are less than the mean and some are greater. We aren't necessarily concerned about above or below at this point, but how far the deviations are from the mean. We can capture this by squaring the deviations from the mean, and then adding them up, $\sum_{i=1}^{N}(x_i - \bar{x})^2$. When we divide this by the number of measurements minus one (N – 1), we get a quantity called the "variance", which is sort of like the "average of the squared deviation".

Why not divide by N since we had N measurements? The answer is beyond the scope of this introduction but has to do with degrees of freedom in the dataset that are left over after we have determined the mean.

Finally, to relate the variance to the original measurement, we take the square root (to "undo" the effect of squaring the deviations), giving the standard deviation (σ), as shown in Eq. 2. Note that whatever units the original measurement may have had, the standard deviation has the same units! Check Eq. 2 and verify this. You could also try calculating σ for each of the above datasets and show that they are different.

When the dataset is large and the errors involved in the measurement are random (as opposed to systematic), the data is often distributed in a special way called a Gaussian, or normal, distribution. This distribution can be graphed as a probability function, as seen in Figure 1.1, where the probability of any particular measurement is plotted as a function of *x*, the measured value. Note that the peak of the function is at the mean value, $\bar{x}$. This is the most probable value that you would find when making a measurement of a quantity whose results fit a Gaussian distribution. The curve is symmetric about this mean value, falling off gradually on either side and forming a "bell-shaped" curve.

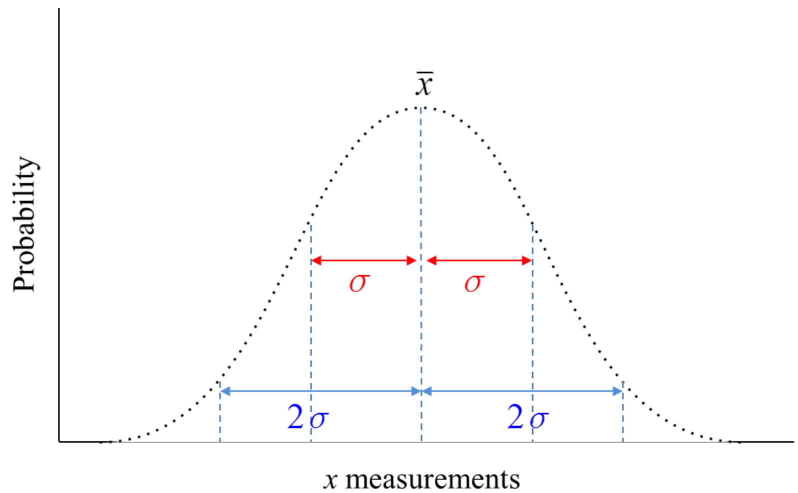

Figure 1.1

Another feature of the curve involves the standard deviation. Recall that the standard deviation has the same units as the measured quantity. If we add or subtract the standard deviation from the mean, we create a range of possibilities for a forthcoming measurement, *x*: $(\bar{x}-\sigma)<x<(\bar{x}+\sigma)$. You can see this range in the figure, indicated by the red arrows. Numerically, when the distribution is Gaussian, there is about a 68% chance that a measurement will fall within this range around the mean. Look at how it is much more probable the measurement is within this range (near the mean) than it is outside this range (because the probability curve is higher in this range). If we look at an extended range for the possibility of a forthcoming measurement, up to 2 standard deviations from the mean, $(\bar{x}-2\sigma)<x<(\bar{x}+2\sigma)$, there is a 95% chance that the measurement will fall within this range.

Note that the shape of the Gaussian distribution can be more narrow around the peak, or even more broad than in Figure 1.1. This indicates differences in the value of the standard deviation. When the standard deviation is big, the curve will necessarily be wider, and when it is small, the curve will be narrower. A larger standard deviation means there is more error in any one measurement, and a smaller standard deviation means there is less error in any one measurement.

One way to view the probability distribution in a dataset without knowing the actual probability of any one measurement is to construct a *histogram*. An example is shown in Figure 1.2. A histogram is very similar in appearance to a bar graph. On a histogram, what you normally think of as the "x-axis" is actually a set of ranges for possible values of the measurement x. The "y-axis" is the number of measurements that occurred in any one particular range. This is like having a row of bins along the "x-axis", each labeled with the range, such as "$0 < x < 2$" and so on. Every time a measurement occurs in one of the ranges defined by a bin, we toss a ball into the bin. When we have completed a large number of measurements, say 50 or 100, we then construct a histogram where the height of a bar for any bin is based on how many balls are inside.

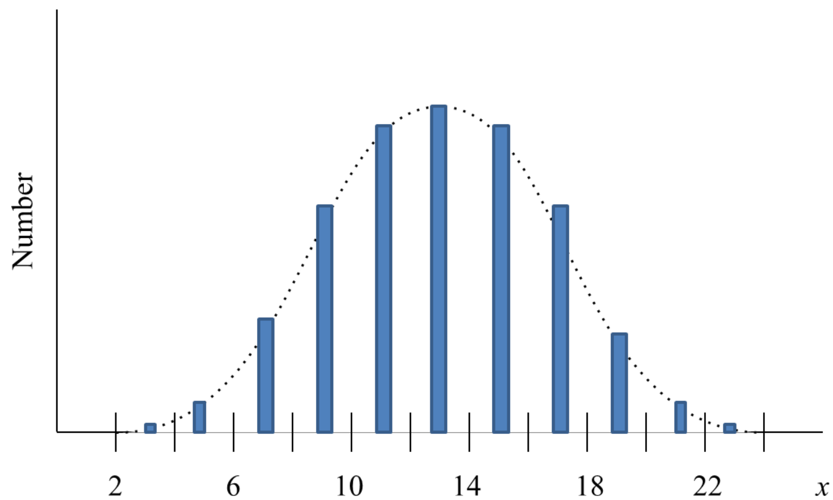

Figure 1.2

The histogram shown in Figure 1.2 is for a distribution that is also Gaussian. We can see this from the overlying Gaussian curve that is shown to coincide with the frequency of the measurements in any one bin. In the figure, the bins were chosen to have a range of "2". For example, the first bin is from 0 to 2, the second is from 2 to 4, and so on. Making a histogram is a good way to visualize the distribution of your data and can give you a snapshot to see if the profile appears to be Gaussian. Note that the bin size could be made smaller to give us finer detail, or made larger. This would change how many bars we had in the histogram, but the general shape of the profile would remain unchanged.

Recall that we earlier wondered how confident we could be in our measurements, or in the mean value that we derive from these measurements. The *standard deviation of the mean*, σ_m, is a quantity that gives us some idea of how confident we should be in the *mean* derived from our measurements. Naturally, this depends on the measurement error or *standard deviation*, and the relationship between the two is defined in Eq. 3, where N is the number of measurements.

$$\sigma_m = \sigma / \sqrt{N} \qquad \text{(Eq. 3)}$$

An interesting fact to note here is that the standard deviation indicates the average measurement error, so it will not change very much as the sample size changes. But the standard deviation of the mean will decrease as the number of measurements increases! This means that we are becoming more and more confident that the mean which we may have calculated earlier with fewer measurements, really is the mean of the dataset. If we continue to measure events, our certainty will only grow as the standard deviation of the mean continues to decrease.

What the *standard deviation of the mean* allows us to say is that we are 68% certain that, if more measurements are made, the new value of the mean ($\bar{x}_{new}$) will be between $\bar{x}-\sigma_m$ and $\bar{x}+\sigma_m$, where $\bar{x}$ is the current mean. We are also 95% certain that, if more measurements are made, the new value of the mean will be between $\bar{x}-2\sigma_m$ and $\bar{x}+2\sigma_m$. As the standard deviation of the mean decreases with increasing trials, these ranges necessarily shrink, so that the 95% confidence region gets tighter and tighter.

Here, we will use the total on the rolling of two six-sided dice as a random event to be measured and statistically analyzed. Analyzing over a different number of trials will allow us to see if the standard deviation and mean remain roughly constant, while the standard deviation of the mean decreases with more trials. Histograms of the dice rolls will also be created to visualize the results. We will also attempt to use the distance a meter stick falls as a way of measuring our reaction time, statistically analyze the results, and create a histogram for each lab partner's results.

<u>Goals</u>: (1) Perform statistical analyses investigating the *mean*, *standard deviation*, and the *standard deviation of the mean*.

(2) Investigate whether or not the standard deviation and the standard deviation of the mean remain constant, increase, or decrease as the number of trials increases.

(3) Become familiar with creating a histogram.

Procedure

Equipment – computer with Microsoft Excel, ruler or meter stick, two six-sided dice

First, we will generate two sets of data with the dice for analysis in the next section

1) Have one partner roll the dice and add the number of dots shown on the sides that are face-up. For example, the total for the dice on the table in the picture below would be 10 = 6 + 4.

The other partner should **record** the result of the roll. Repeat this process until this partner has rolled the dice 40 times. **Record** the results for each of the 40 rolls. You may want to organize these in a column, or table. This data set of 40 rolls will be referred to as "Set A".

HINT: Rather than writing the data out longhand, you may prefer to type the data directly into an Excel spreadsheet. If so, see the instructions in the first three paragraphs of the Data Analysis section below.

2) Have the other partner roll the dice 40 times, while the previous roller **records** the results of each roll. This dataset of 40 rolls will be referred to as "Set B".

We will now attempt to gauge the reaction time for you and your partner. In reality, what we will measure is the distance a ruler (or meter stick) falls before you are able to catch it. This will again involve a measurement that each of you should perform 40 times. Each measurement should be conducted in the same way as follows:

3) Have your partner hold the ruler (or meter stick) vertically in the air so that its lower end is about even with your chest. Hold out your hand and, without touching the ruler, put your index finger and thumb on either side of the ruler so that you will be able to pinch the ruler (catch it) when it falls. You should have the bottom of the ruler even with the top of your thumb and finger (see the image below). In this way, when you pinch and catch the ruler, you will look at where the top of your thumb and finger grab the ruler to measure how far it has fallen (see the image below).

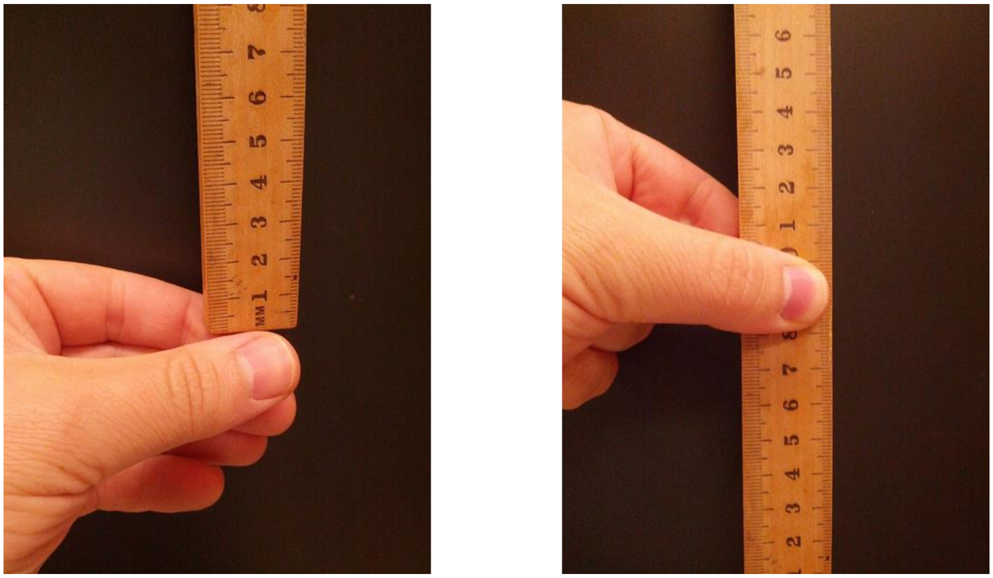

Figure 1.3

4) Once you have arranged the ruler even with the top of your thumb and finger, have your partner drop the ruler without warning you. You should pinch the ruler as soon as you can after it is released. **Record** the distance the ruler has fallen (in cm – and remember that you should be able to estimate to the tenth of a cm on your ruler!). This should be the length of the ruler from its lower end to the top of your pinched thumb and finger. If your reaction time is slower, the distance will be greater because the ruler will have fallen farther.

5) Repeat this process until you **record** a total of 40 trials. You should both refer to this dataset as "Set C".

6) You and your partner should now switch roles and repeat steps 3 through 6. **Record** the 40 trials for your partner. You should both refer to this dataset as "Set D".

As always, be sure to organize your data records for presentation in your lab report, using tables and labels where appropriate.

<u>Data Analysis</u>

The following instructions are intended to aide you in using Microsoft Excel to perform the following calculations. You should also consider other organization techniques for presentation of your data, or results, though printing the results from the instructions here is a good start. You could, for example, choose to make the font in the cells for each dataset a different color.

Open a new worksheet in Excel. Refer back to the Introduction to Excel lab for some Excel basics if necessary. Click on the cell “A1” and type “A”. Click on the cell “B1” and type “B”. Click on the cell “C1” and type “C”. Click on the cell “D1” and type “D”. Then hit “enter” (or “return”).

Now, enter all of your results for Set A in the cells A2 through A41. Enter all of your results for Set B in the cells B2 through B41. Enter all of your results for Set C in the cells C2 through C41. Finally, enter all of your results for Set D in the cells D2 through D41. We now have all of the data entered into the spreadsheet and each column is appropriately labeled.

We will now calculate the mean for each data set. Click on cell “A43” and type “=SUM(A2:A41)/40”. Then hit “enter” (or “return”). What is this function doing? It is calculating the average of your values in Set A. It is summing all of the values and dividing by how many there are (40). Type a similar equation in the cells “B43, C43, and D43” to get the mean for those data sets. Confirm that these calculations are working correctly by calculating the mean for Set A by hand, using Eq. 1 and compare to the result in Excel. Are your results in agreement?

Click on the cell “E43” and type “mean” and then hit “enter” (or “return”). This way, we will know that the numbers in row 43 those are the mean values of each column.

We now want a formula to find the standard deviation for each dataset. To do this, we first need to get the “difference from the mean – squared” for each measurement in each data set. We will choose a location away from our organized data to do this. Click on the cell “A50” and type “Asd”. Click on the cell “B50” and type “Bsd”. Click on the cell “C50” and type “Csd”. Click on the cell “D50” and type “Dsd”. This way, we have labeled the columns for each dataset, but signified this is for a finding the standard deviation by the labels we have chosen.

Note that we are now working on the numerator of Eq. 2. We will detail the operations for Set A. You should then repeat these operations for Sets B, C, and D. Just follow the same set of instructions and anywhere you see an “A”, just change it to the appropriate letter for your dataset.

Click on the cell “A51” and type “=(A2-A43)^2”. Then hit “enter” (or “return”). Click again on the cell “A51” and locate the small square in the lower right corner of the cell. Carefully click on that small square and hold the mouse button down. Drag the cursor downwards until you reach cell “A90”. Then, let go of the mouse button. This should have repeated a similar formula in all of those cells. The “$” in the formula are important. They keep the name of the cell “A43” from changing as we move from one cell to the next. Click on the cell “A52” and look at the formula in the value bar just above the spreadsheet. It should say “=(A3-A43)^2. It is taking the difference between the second data value and the mean and then squaring the result. This formula has taken each value in the data set, subtracted the mean, and then squared the result. If you look at Eq. 2, this is what we need to get the numerator.

In order to finish Eq. 2 for Set A, we now need to sum the results in the cells “A51” through “A90”, divide by $N - 1$ (i.e., 40 – 1 = 39), and then take the square root of the result. To get the standard deviation for Set A, click on cell “A44” and type “=SQRT(SUM(A51:A90)/39)”. Then hit “enter” (or “return”). This performs the sum, divides by 39, and then takes the square root. We have completed Eq. 2!

Repeat the entire process for finding the standard deviation for the datasets B, C, and D.

Click on cell “E44” and type “standard deviation” to label the results appearing in row 44. We should now have the mean and standard deviation for each data set.

We now want the standard deviation of the mean for each dataset. Click on cell “A45” and type “=A44/(SQRT(40))”. This is the same as Eq. 3, is it not? The result is the standard deviation of the mean for Set A.

Repeat the last step, making proper adjustments to the formula to get the standard deviation from the mean for the other 3 datasets in cells “B45, C45, and D45”.

Then, click on cell “E45” and type “standard deviation of the mean” to label the results appearing in row 45. Then hit “enter” (or “return”).

At this point, you have computed statistics for several data sets (mean, standard deviation, and standard deviation of the mean) by hand, in the pre-lab questions, and in the series of calculations, above. Excel has functions that calculate the mean and standard deviation. Let’s practice using them, too.

Click on cell “A46” and type “=AVERAGE(A2:A41)”. When you hit “enter” (or “return”), it should show the mean of the values in cells “A2” through “A41”. Similarly, enter into cell “A47” the equation for the standard deviation: “=STDEV(A2:A41)”. Unfortunately, Excel does not have a function for the standard deviation of the mean, but the calculation is easy.
In cell “A48”, type in “=A47/(SQRT(40))”.

Repeat the last steps, making proper adjustments to the formulas to get the three statistics for the other three datasets in columns “B, C, and D”. Then, add labels (“Excel mean”, etc.) in column E. Compare the values from the Excel functions (AVERAGE and STDEV) with their counterparts in rows 43, 44, and 45 that you calculated using Eq. 1, 2 and 3. Are they the same?

HINT: In many, perhaps most, of the Experiments you will do in lab throughout the rest of the semester, you will make repeated measurements of some quantity, then calculate the mean, standard deviation, and standard deviation of the mean. The "best estimate" of a quantity stated with its accuracy is usually written as the mean ± the standard deviation of the mean (for example, the length of a shark is 2.16 ± 0.15 m). You can use the functions AVERAGE and STDEV in Excel to make these calculations. In important cases, you may want to draw a histogram and include it in your lab report to visualize the data. *Most scientific calculators also have a Statistics Mode with button sequences to compute these statistics; you may use them, too.* See the instructions for your particular calculator or ask a TA for help.

Question 1: Consider the mean, standard deviation, and standard deviation of the mean for Set A and Set B (the dice throws). Should the mean and standard deviation be very different for each set, or should they be nearly the same? What about the standard deviation of the mean? Explain.

Question 2: Consider the mean, standard deviation, and standard deviation of the mean for Set C and Set D (the reaction measurements). Who reacted quicker and why? How can you tell? Was one person more consistent in reacting than another? Again, how can you tell? Explain.

Question 3: Consider the mean, standard deviation, and standard deviation of the mean for Set C and Set D (the reaction measurements). Subtract one mean from the other to get the difference in the means. Given the standard deviations of the mean for each case, and this difference you have calculated, can we say for certain that one person reacted faster than another? Explain.

We will now analyze the dice data a little further by creating a histogram for each set (A and B). Before we start, you might want to review the discussion of the histogram in the introduction, or read about it from another source.

TA: Before class, you may want to print out copies of the histogram page of this lab. Make enough for all the students in lab, plus ~20% extra in case students make errors and redo. Hand them out during lab.

Our histogram will present the number of times in Set A that we rolled a "2", the number of times we rolled a "3", etc., up to "12". The empty histogram plots on Figure 1.4 are prepared to accept these counts for Set A and Set B. Let's prepare a column in Excel for the bin centers: in cells "F2" to "F12", type in the sequence of numbers, 2, 3, 4, …, 12 that represents the range of possible numbers on a roll of two dice. Label this column in cell "F1" with "Bin".

Now we must count up the number of times in Set A that we rolled a "2", etc. You can do this manually by scanning down the column "A2:A41" and counting the number of "2's", and entering this into cell "G2". Now count up the number of "3's", "4's", etc., up to "12's", and enter them into the corresponding cells in "G3:G12". In lieu of doing the counting yourself, you can use a function in Excel to do the counting: in cell "G2", type "=COUNTIF(A2:A41,2)". This counts the number of times a "2" appears in the cell range "A2:A41". Repeat the COUNTIF command in cells "G3:G12" keeping the count range in column A the same, but changing the "IF" number from 2 to 3, 4, etc.

It is good to get in the habit of "building in" steps that confirm your work. Here, we will confirm that we correctly counted all the dice rolls in cells "A2:A41" by summing the counts in cells "G2:G12". In cell "G13", enter the equation "=SUM(G2:G12)". It should equal 40 (the number of rolls). If it does, you can continue with confidence; if not, you should backtrack to find the error.

We are nearly done with Set A. All we have to do now is draw the histogram in the part of Figure 1.4 labeled "Set A". Start by plotting the counts (cells "G2:G12") as a function of bin number (cells "F2:F12") on the *y*- and *x*-axes, respectively, as you would in a normal scatter plot. Rather than connecting the points, make vertical histogram bars, like those in Figure 1.2, that go from the *x*-axis up to each point.

HINT: Use a ruler to make your histogram bars neat and square. Shade them in if you like.

Now, repeat the histogram-building steps using the data from Set B (that is, cells "B2:B41") and entering the counts in each bin into cells "H2:H12".

On the histogram for Set B, mark the positions of the mean, the mean ± σ, and the mean ± 2σ, as shown on Figure 1.1, using the values you calculated in your spreadsheet. You can also draw the positions of the mean ± σ_m, to show the range of positions where the "true mean" probably lies (there is a 68% chance that it lies within this range).

Get in the habit of labeling things in your Excel spreadsheet. Here, it is appropriate to label the columns "G2:G12" by typing "Set A" into cell "G1". Repeat with appropriate changes for the counts from Set B.

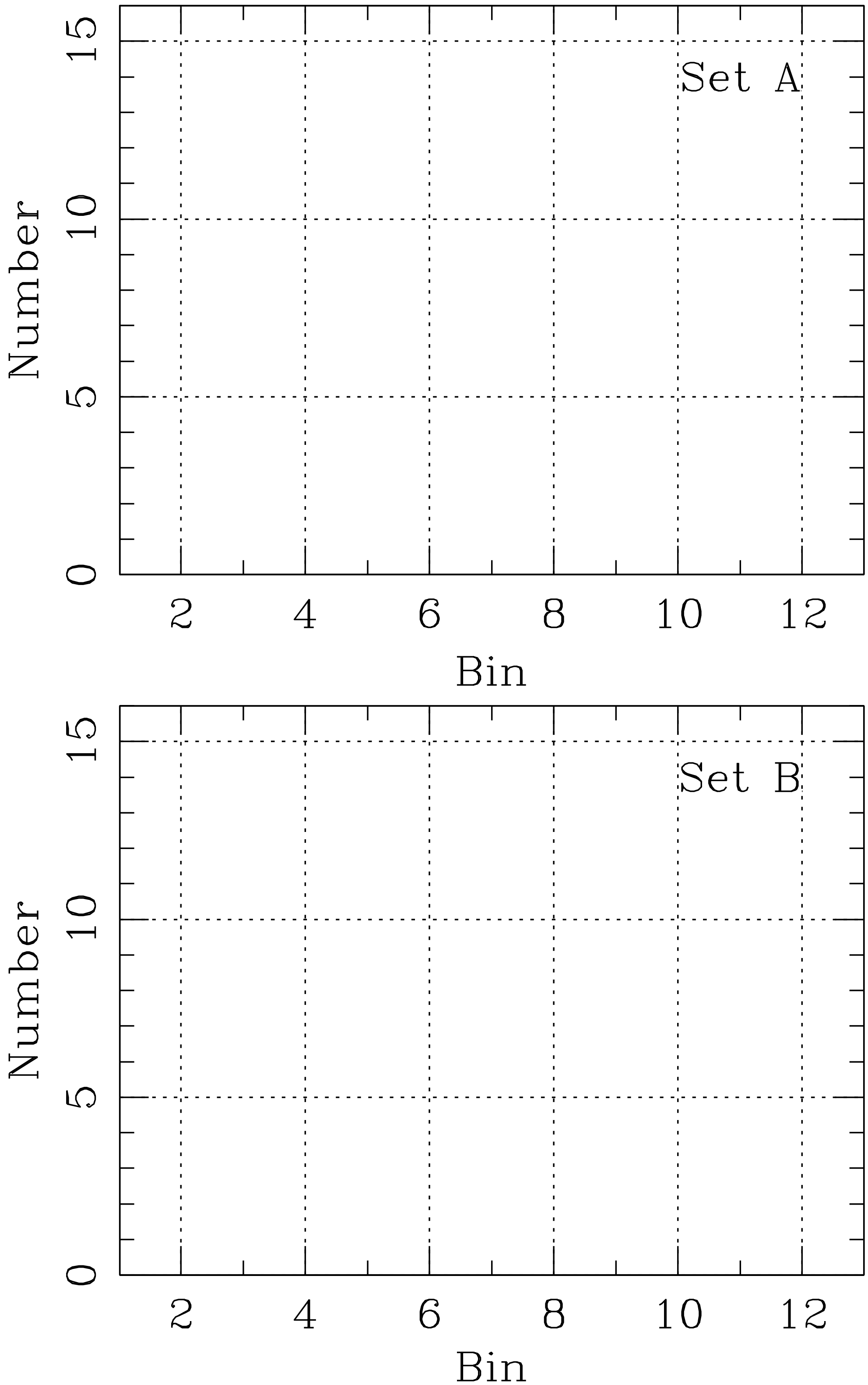
Set A
Number
0
5
10
15
2
4
6
8
10
12
Bin
Set B
Number
0
5
10
15
2
4
6
8
10
12
Bin

Figure 1.4

Question 4: Do the histograms appear to represent a Gaussian distribution for the dice throws? Should they appear similar to a Gaussian distribution? Explain why or why not.

HINT: If you remember combination theory and how combinations combine from past math courses, you should be able to calculate the probability of any particular roll, say 1+1=2 or "snake eyes." The probability of each combination can be used to make an overall probability histogram that can be compared with the observed histogram for Set A. Statisticians talk about the theoretical, "underlying" probability function, and the "realized" data set drawn from it by an experiment like rolling the dice 40 times.

Error Analysis

We have actually already done a lot of error analysis in this lab. Here, we will test the idea that the standard deviation of the mean decreases with more trials, while the standard deviation stays roughly the same.

To do this, we can use a feature of Excel that allows us to access non-consecutive cells. In cell "G14", type "=AVERAGE(A2:A41,B2:B41)" to compute the mean of the 80 cells in Set A plus Set B, combined. Label it in cell "H14" with "Mean80".

Calculate the standard deviation of the 80 values by typing "=STDEV(A2:A41,B2:B41)" in cell "G15", and labeling it with "SD80" in cell "H15".

Finally, compute the standard deviation of the mean of the 80 values by typing "=G15/(SQRT(80))" in cell "G16", and labeling it with "SDoM80" in cell "H16".

HINT: Notice that colored boxes surround the spreadsheet cells that are selected in an equation. It is helpful to check these boxes when selecting cells to confirm your selections.

Question 5: Consider the standard deviations and standard deviations of the mean for the individual sets A and B and of the combined set we just calculated. Did the standard deviation change or stay roughly the same when we increased the trial size from 40 to 80? Did the standard deviation of the mean change or stay roughly the same with the increased number of trials analyzed? Do these results match your expectations? Explain what these results mean. You might reread the introduction to aid you in answering this.

TA: You may want to try this extensional activity: Ask your students to email you the Excel spreadsheet from each lab group. Cut/paste the dice-roll data in Columns A and B into a single spreadsheet, and compute the mean, standard deviation, and standard deviation of the mean for this very large sample (as large as {10 labs} * {80 rolls} = 800 rolls of the dice!). Share the statistics with the students at the beginning of the next lab meeting to further demonstrate how increasing the number of trials affects the statistics. You might also draw a histogram and display it, to show how the histogram becomes smoother as the number of trials increases!

Questions and Conclusions

Be sure to address Questions 1-5 and describe what has been verified and tested by this experiment. What are the likely sources of error? Where might the physics principles investigated in this lab manifest in everyday life or in a job setting?

Pre-Lab Questions

Please read through all the instructions for this lab to acquaint yourself with the experimental setup and procedures, and develop any questions you may want to discuss with your lab partner or TA before you begin the lab. Then answer the following questions and type your answers into the Canvas quiz tool for this lab, Statistical Analysis of Data, and submit it before the start of your lab section on the day this lab is to be run.

PL-1) *Imagine this:* Bob the Builder is measuring the size of a room so he can buy the right amount of hardwood flooring. He wants to be precise, so he does not buy too much or too little. Unfortunately, he forgot his tape measure and only has a meter stick with him. Bob uses the meter stick to measure the length of the room at the East wall to be 6.28 m. Because he has to move the meter stick along the floor several times to cover the full length of the wall, he knows there will be measuring error, so he measures it twice more, obtaining 6.30 m and 6.25 m. Thus, he has a data set with three values: {6.28 m, 6.30 m, 6.25 m}. Calculate the mean value of this data set; report the answer in meters.

PL-2) Calculate the standard deviation of Bob's data set, {6.28 m, 6.30 m, 6.25 m}. Do it by hand to get a good sense of how the equation works. Then, if you know how to, you can confirm it using the statistics functions on your calculator or Excel.

PL-3) Bob measures the West wall of the room four times, and calculates the mean of his data set to be 6.293 m and the and standard deviation to be 0.032 m. Calculate the standard deviation of the mean (in meters) for this data set.

PL-4) When you have two or more measurements in your data set ($N \geq 2$), which of the following statement(s) is/are correct?

(A) The standard deviation is always greater than or equal to zero, $\sigma \geq 0$, (B) The standard deviation of the mean is always greater than or equal to zero, $\sigma_m \geq 0$, (C) The standard deviation is always larger than or equal to the standard deviation of the mean, $\sigma \geq \sigma_m$, (D) Answers *A* and *B*, (E) All of answers *A*, *B*, and *C*.

- A
- B
- C
- D
- E

PL-5) The quantity that best describes our confidence in our experimental mean values, or establishes a confidence interval for our mean is

- (A) the standard deviation.
- (B) the average.
- (C) the standard deviation of the mean.
- (D) the arithemetic mean.
- (E) the number of measurements.