Uniform Circular Motion

Goals and Introduction

The purpose of this experiment is to investigate the scenario of an object in uniform circular motion. An object is in *uniform circular motion* when it travels in a circular path with radius, *r*, and moves with a constant speed, *v*. Though the object moves with a constant speed, its velocity changes because the direction the object is moving is constantly changing. (Remember that speed is the magnitude of the velocity.) By definition, if the velocity of an object is changing, it is being accelerated. Thus, there must be an acceleration responsible for changing the direction of an object's velocity when it is in uniform circular motion. A graphical examination of the change in velocity for an object moving in a circle is shown in Figure 10.1.

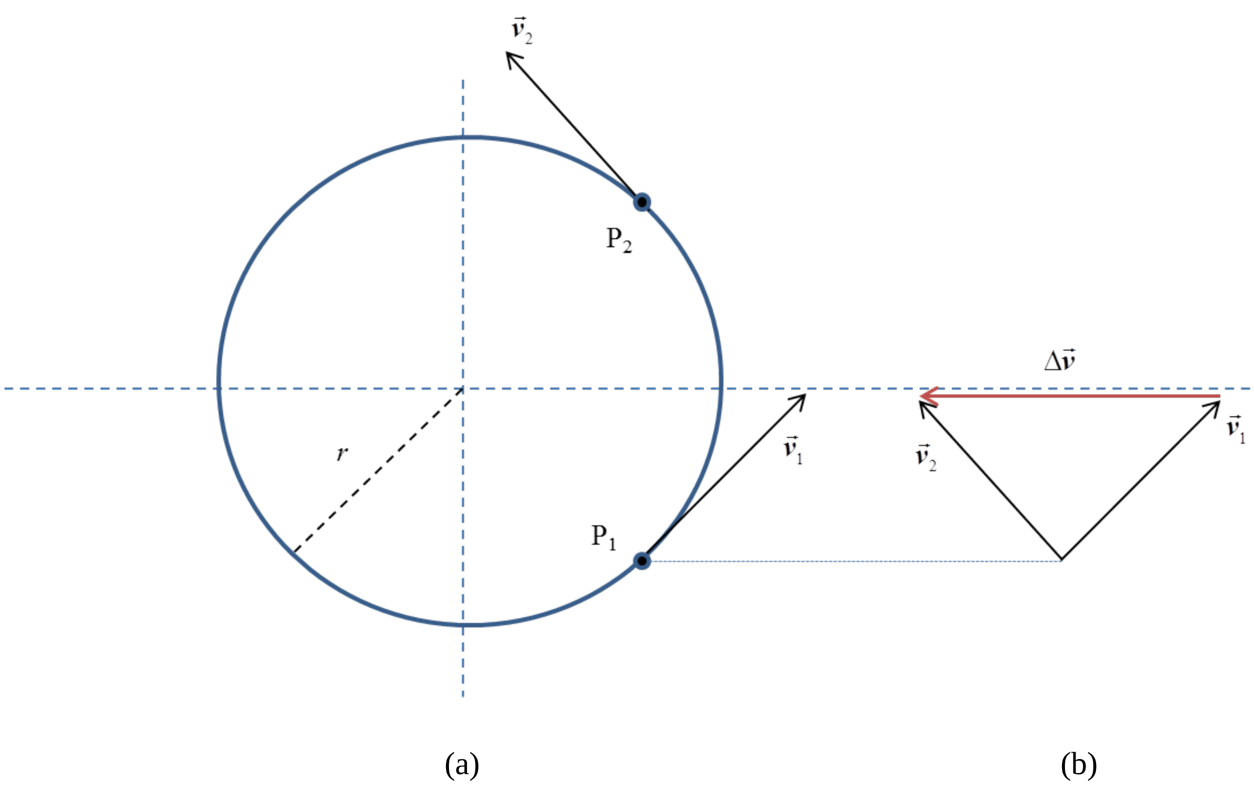


Figure 10.1

At any instant, the velocity of an object in circular motion is directed tangent to the circular path. This is illustrated by the vectors $\vec{v}_1$ and $\vec{v}_2$ in Figure 10.1(a), which represent the object's velocity when it is at the locations P_1 and P_2 respectively. By definition, the direction of the change in velocity is the same as the direction of the acceleration of the object, as in the equation,

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}. \quad \text{(Eq. 1)}$$

We can find the direction of the change in velocity graphically by recognizing that $\Delta\vec{v} = \vec{v}_2 - \vec{v}_1$ between points P_1 and P_2. Recall that we can find the vector sum of two vectors graphically by adding them using the *Tip-to-Tail Method*. To make use of this in this instance, it may help to rearrange our equation for the change in velocity to appear as $\vec{v}_1 + \Delta\vec{v} = \vec{v}_2$. This says that there is some vector, $\Delta\vec{v}$, that can be added to the vector $\vec{v}_1$ and result in the vector $\vec{v}_2$. In the tip-to-tail method (see Figure 10.1(b)), the tail of the second vector in the sum is placed at the tip of the first vector, and the resulting vector is drawn from the tail of the first vector to the tip of the second vector. Knowing the velocity is tangent at the two points, we can complete the first vector and the resulting vector in this relationship. In other words, the first vector in the sum, $\vec{v}_1$, can be drawn over to the right, away from the main circle, and the resulting vector, $\vec{v}_2$, can be drawn, starting from the tail of $\vec{v}_1$. By connecting the tip of $\vec{v}_1$ and the tip of $\vec{v}_2$, we draw the vector $\Delta\vec{v}$, which is what must be added to the first velocity to result in the second.

Note that the direction of this vector, $\Delta\vec{v}$, points towards the center of the circular motion. In fact, if we repeated this process at other locations, we would again find that the necessary change in velocity points towards the center of the circular motion. As stated in Eq. 1, this means there must always be an acceleration towards the center of the circular motion. Again, this acceleration is not changing the speed of the moving object, but is constantly changing the direction. For this scenario (an object moving in a circular path), we give a special name to this necessary acceleration – *the centripetal acceleration*, a_c.

Imagine swinging an object in a circle that is attached to a string in your hand. You may even try this using some spare string, or a shoelace, and a small object that could be attached to the string (Though be careful to not poke out your eye! Physics can be dangerous. ☺). Suppose that you get the object moving in a circle and with a constant speed. Here, the tension in the string is a force that is always pulling towards the center of the motion. It is responsible for changing the direction of the object, and thus it is a force responsible for the centripetal acceleration, a_c, in this scenario.

There are a few ways we can quantify the uniform circular motion of this object. First, the object is traveling a specific distance again and again - the circumference of the circular path ($2\pi r$). Since the object is traveling with a constant speed (v), the time it takes to travel the circular path is the same each time. The time necessary to complete one cycle is called *the period* (T). Since the speed of an object is the distance it travels divided by the time,

$$v = \frac{2\pi r}{T}. \quad \text{(Eq. 2)}$$

Thus, if we measure the length of the string from our hand to the center of mass of the object, and measure the time it takes for the object to travel the circular path while we swing it, we could calculate the speed of the object.

Now suppose you try to spin the object faster, while keeping the length of the string (and thus the radius of the circular path) constant. What do you have to do to accomplish this? You probably have to rotate your hand more quickly and perhaps have a firmer grip on the string. In essence, you are pulling harder on the string, and thus on the object.

Why should you have to pull a little harder to make the object travel at a greater speed on the same circle? Think again about our vectors in Figure 10.1(b). If the speed is greater, then the length of the velocity vectors would be greater, though they are on the same-sized circle. If you think through the vector addition again, the vector that must be added to $\vec{\boldsymbol{v}}_1$ in order to result in $\vec{\boldsymbol{v}}_2$ is still the vector $\Delta\vec{\boldsymbol{v}}$. However, $\Delta\vec{\boldsymbol{v}}$ will need to be longer than it was previously (it still points towards the center). This means that its magnitude must be greater, which means the centripetal acceleration must be greater! So, the speed of an object plays a role in determining the amount of centripetal acceleration.

A similar effect can be observed if you shorten the length of the string between your hand and the object and try to spin it at the same speed as when the string was longer. Given a particular speed, it takes more centripetal acceleration to travel a smaller circular path, because the velocity vector needs to be turned more quickly. It can be shown mathematically that the proper relationship between these quantities is given by

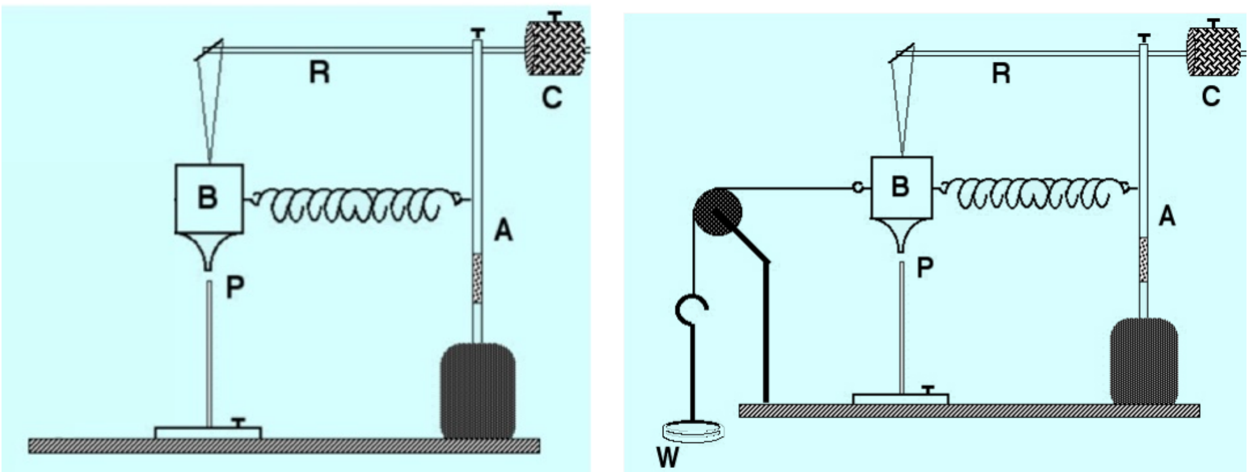
$$a_c = \frac{v^2}{r} \quad \text{(Eq. 3)}$$

where, a_c is the centripetal acceleration, v is the speed of the object, and r is the radius of the circular path.

Thinking about our scenario of an object attached to a string, the force responsible for causing the centripetal acceleration is the tension in the string. In other scenarios, a different force may responsible for causing the centripetal acceleration. In any scenario, though, Newton's second law can be used to describe the amount of force causing this centripetal acceleration on an object with mass, m. We call this force the *centripetal force*, F_c, where

$F_c = ma_c$. (Eq. 4)

Like the centripetal acceleration, the centripetal force must point towards the center of the circular motion. Note that this IS NOT a new type of force or source of force. We are merely using Newton's second law to say that if you know a certain mass is subject to an amount of acceleration, it is being caused by some amount of net force, F_c. In every case of an object moving in a circle, there is some *real* force (or a net amount of force) that is acting as the centripetal force. For example, the tension was acting as the centripetal force in our scenario of an object attached to a string. For the case of the Moon orbiting the Earth, the gravitational force is always pulling the Moon toward the Earth and acting as the centripetal force.



(a) (b)

Figure 10.2

Today, we will use the apparatus shown in Figure 10.2(a) to measure the centripetal force necessary to keep a mass whirling in a circular path at constant speed. The apparatus consists of a bob **B** suspended by string on a support rod **R** that is attached to a rotating axle **A**. There is also a pointed tip, **P**, that will be used as a target location for the bob, as it spins around the axle **A**. The bob is attached to the axle by means of a spring that provides the centripetal force necessary for uniform circular motion. The position of the bob along the rod, and therefore its distance from the axis of rotation, is adjustable. The other end of the rod has a counterweight **C** that can also be adjusted so as to maintain rotational stability (avoid wobble). The assembly can be rotated about the vertical axis, through **A**, by using your fingers. The quality of the bearings is so good that it is quite easy to maintain a constant speed, and thus experiment with uniform circular motion.

The objective is to rotate the bob at a constant speed so that its tip lines up with the top of the pointer **P**. When this happens, you will be able to measure the radius, r, of the circular path, and using a stopwatch, determine the period of the circular motion, T. Because it may be difficult to time the bob for one trip around the circle, it is best to find the total time, Δt, for a number of repetitions or cycles, N, and calculate the average time for one cycle. The period is then

$$T = \frac{\Delta t}{N}. \quad \text{(Eq. 5)}$$

Once the radius and period are known, the speed and centripetal acceleration can be calculated. Measuring the mass of the bob, the centripetal force can also be determined, and this entire process may be repeated for different starting distances of the bob/pointer from the axle.

In order to check our calculated results of the centripetal force from the circular motion analysis, we can measure the force exerted by the spring by stretching it the same amount as when it was moving in a circle. This is accomplished by determining the amount of weight, at **W**, that is necessary to achieve the proper stretch of the spring, as depicted in Figure 10.2(b). This is not done while rotating the apparatus, but afterwards. Think about the free-body diagram of the bob (not moving) when it has the spring force pulling to the right and the tension attached to the weight pulling to the left. Why is the amount of weight at **W** the same amount of force being exerted by the spring?

Goals: (1) Investigate and quantify aspects of circular motion.

(2) Calculate the required amount of force to keep an object in circular motion by using measurements of aspects of the circular motion (speed, period, mass, radius).

(3) Separately, measure the amount of force that was being exerted during the circular motion directly, and compare to the results from the circular motion analysis.

Procedure

Equipment – meter stick (or other distance-measurement tool), mass holder with removable masses, balance, stopwatch, the bob-rod-counterweight apparatus, manual rotating axle with spring, pulley

1) Remove the bob, **B**, from the apparatus and **measure and record** its mass, m, using the balance.

2) Attach the bob, **B**, to the support rod, **R**, and adjust its position so that it is located as close to the rotating axle, **A**, as possible. Notice that there is a flat region on the support rod where the thumbscrew can clamp it. DO NOT go beyond the flat portion on the rod. Make sure that the rod is clamped securely.

3) Adjust the position of the counterweight, **C**, so that it is located as close to the axle, **A**, as possible. Again, DO NOT go beyond the flat portion on the support rod.

4) Adjust the pointer, **P**, so that it is accurately lined up with the tip of the bob. **Measure and record** the distance from the pointer to the center of the axle. This will be the radius, r, of the circular motion of the bob.

5) Now, attach the spring to the bob and the axle support.

6) Practice rotating the axle, **A**, at a constant rate so that the bottom tip of the bob, **B**, passes directly over the pointer, **P**. When the bob moves in this way, it is traveling in a circle with a radius equivalent to what you recorded in step 4. The spring is pulling on the bob throughout the motion, and the spring force is acting as the centripetal force. In order to measure the period, one person will need to operate the stopwatch while the other person rotates the axle. Because it may be difficult to time the bob for one trip around the circle, it is best to find the total time, Δt, for a number of revolutions, N, and calculate the average time for one revolution. The person using the stopwatch will also need to count the number of revolutions. Practice this process a few times before proceeding to the next step.

7) **Choose and record** a number of revolutions, N, that you think is appropriate for determining the period based on your practice rounds.

HINT: One revolution is not sufficient. Your reaction time will be significant compared to the period you measure. When you spin the axle with the bob passing over the pointer, it will go through several revolutions before slowing down and drifting towards the center. Get an idea of a good value for N that you could use by practicing the measurement process.

8) Then, while one partner spins the axle so that the bob continues to pass over the pointer, the other partner should use the stopwatch to **Measure and record** the total time, Δt, for the bob to travel through the number of revolutions you chose. Repeat this process four more times so that you have five measurements.

9) After you stop rotating the axle, Attach the mass holder, **W**, to the bob by running a cord over the pulley. The spring should still be attached to the other end of the bob.

10) Add mass to the mass holder, **W**, until the bottom tip of the bob is aligned with the pointer, **P**. Notice that the spring is stretching! When the bob is now hovering over the pointer, it is in equilibrium. The tension in the cord must be equal in magnitude to the spring's restoring force. Likewise, the weights are in equilibrium, so the tension in the cord must be equal in magnitude to the gravitational force (weight) on the masses at **W**. How does the spring force compare to the gravitational force on the total mass at **W**? Also, think about what direction the spring force is acting upon the bob, compared to the tension in the cord.

11) Use the balance to **measure and record** the total mass, M, that brought the bob in line with the pointer. Don't forget to include the holder itself, since it has mass and was also hanging!

12) Repeat steps 2-11 three more times, but move the bob and pointer about 1 cm further away from the axle each time. You will also need to move the counterweight, **C**, about the same distance from the axle each time in order to maintain rotational stability.

13) Before proceeding to performing an analysis of the experiment, it might be wise to organize your data especially, since you will have multiple time measurements for each position of the pointer.

Data Analysis

Here, we will step through the analysis for one trial (steps 2-11). You should repeat the analysis for the other three positions of the pointer.

For the first position of the pointer, you have recorded five measurements of the total time, Δt, for a certain number of revolutions, N. Calculate the average time for this number of revolutions.

Question 1: Why did we make five time measurements and average them, rather than making just one measurement? What is/are the advantage(s) of this?

Calculate the period, T, for this trial using Eq. 5 and the average total time you found.

Using this period and the radius, r, calculate the speed of the bob, v, using Eq. 2.

Use Eq. 3 and 4 to calculate the magnitude of the centripetal force, F_c, that was acting on the bob while it was in circular motion, where m is the mass of the bob.

Lastly, use the total mass, M, that was added at **W** to stretch the bob and spring to reach the pointer to calculate the amount of the spring restoring force, F_s, when stretched to that position. The logic here was that given equilibrium for the entire system,

$$F_s = F_g \ ,$$

where F_g is the gravitational force, or weight on the total mass, *M*.

Question 2: Why are they equal? Consider the entire system in equilibrium, as depicted in Figure 10.2(b). Clearly explain why the spring's restoring force should be equal to the gravitational force, or weight, hanging from the cord and draw a free-body diagram to accompany your answer.

Repeat this entire analysis (except for Question 1 and 2) for the other three trials (other three distances between the pointer and the axle). It may be wise to organize all of your results in a table with clear labels.

Error Analysis

Ideally, the spring restoring force that you found in each trial should be equal to the centripetal force that you calculated in each trial. However, things are rarely ideal.

Consider the spring restoring force you found in each trial to be the "accepted value", and calculate the percent error between that and the centripetal force you found in each trial.

Question 3: Evaluate your percent error calculations. Does your percent error suggest that the centripetal force and the spring restoring force were identical?

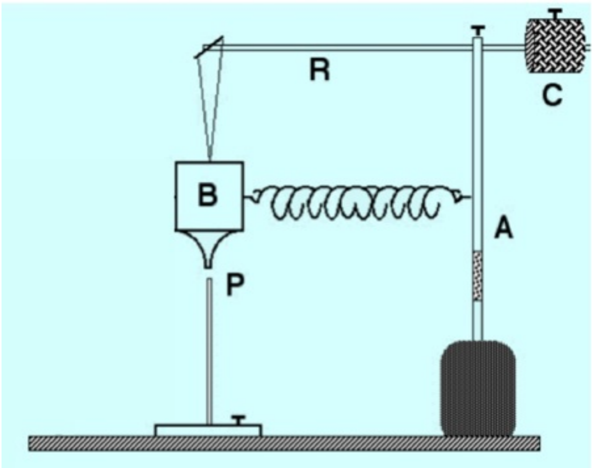
Questions and Conclusions

Be sure to address Questions 1-3 and describe what has been verified and tested by this experiment. What are the likely sources of error? Where might the physics principles investigated in this lab manifest in everyday life, or in a job setting?

Pre-Lab Questions

Please read through all the instructions for this experiment to acquaint yourself with the experimental setup and procedures, and develop any questions you may want to discuss with your lab partner or TA before you begin. Then answer the following questions and type your answers into the Canvas quiz tool for "Uniform Circular Motion," and submit it before the start of your lab section on the day this experiment is to be run.

PL-1) Jane is experimenting using the apparatus for this experiment. In the figure below, we see a “snapshot” of the apparatus at one instant during its rotation. What is the direction of the centripetal force and acceleration vectors of the bob (**B**)?



- (A) Up,
- (B) Down,
- (C) To the left,
- (D) To the right,
- (E) Into the page,
- (F) Out of the page.

PL-2) If Jane spins the axle (**A**) faster, what will happen to the position of the bob (**B**) with respect to the pointer (**P**) compared with the case in PL-1 shown in the figure? Why?

- (A) It stays the same, because the system is in equilibrium,
- (B) The bob will swing in a wider circle, passing to the left of P, because the spring stretches a little and thus provides a larger centripetal force,
- (C) The bob will swing in a smaller circle, passing to the right of P, because the spring stretches a little less and thus provides a smaller centripetal force.

PL-3) Jane uses a stopwatch to measure the time the apparatus takes to complete three full revolutions (she starts and stops the watch as the bob passes the pointer); she measures the time to be 4.2 s. What is the period of rotation in seconds?

PL-4) In a separate trial, Jane determines the period to be 2.14 sec. With the apparatus stopped, she measures the radius to be 13.2 cm and the mass of the bob (**B**) to be 253 g. What was the centripetal force on the bob, in newtons, while it was in uniform circular motion?

PL-5) With the rotation of the apparatus stopped, Jane attaches a string and mass holder (**W**), as shown in the figure. The mass of the mass holder (**W**) itself is 12.8 g. Using the data collected in PL-4 [period = 2.14 s, radius = 13.2 cm, mass of bob = 253 g], predict how much mass (in grams) Jane will need to ***add*** to the mass holder (**W**) in order to stretch the spring so the bob aligns with the pointer (**P**).