

Forces, Vectors, and Equilibrium

Goals and Introduction

As you may have already read in your textbook or heard in class, a basic definition of a force is any “push or pull”. Many pushes or pulls may affect an object at any given moment, as in a tug-of-war (or push-of-war!), resulting in two possible scenarios: (1) the competition of forces results in there being a net amount of force on the object (someone is winning the tug-of-war) and the object accelerates, or (2) the competing forces result in there being no net force on the object (no one is winning the tug-of-war) and the net amount of force is 0. An object subject to forces that falls into this second scenario is said to be in *equilibrium*. Thus, when an object is in *equilibrium*, the net force on the object is 0 and it will not be accelerating. In this lab, you will investigate the forces acting on an object in equilibrium, and use both measurement and vector addition to quantify their relationship to each other.

You are probably more familiar with the English system unit of force – the pound (lb) – than the metric system or SI unit of force – the newton (N). When you hop on a scale and quote your weight in pounds, you are actually quantifying the amount of force with which the scale must push on you to keep you in place. If you wanted to express this force in the SI system, you could use the fact that $1 \text{ lb} = 4.448 \text{ N}$ to convert your weight into newtons. In physics, we develop our equations based on the SI system of units, and so forces should be expressed in terms of newtons.

Now, if the scale and floor weren't there, you would be falling, or pulled downwards. This constant downward force pulling on you is the gravitational force on you due to the Earth. So in this somewhat simple example of you standing on the scale, there are two forces in competition acting on you: (1) the gravitational force pulling downwards and (2) the force of the scale pushing up on you (called the *normal force*, because the force from the contact surface is perpendicular to the surface). It is often helpful to create a pictorial representation of the competition of forces acting on an object, called a *free-body diagram*. A free-body diagram for the scenario of you standing on a scale is shown in Figure 5.1. The object is represented by a dot at the center of the diagram, and each force is then shown as an arrow, pointing in the direction the force pushes or pulls the object. The length of each arrow should be proportional to the magnitude, or amount, of the force it represents, and thus, choosing a scale for the length of the arrows is often helpful in drawing these diagrams.

You are in equilibrium when standing on the scale, and not experiencing a net force, so if these are the only two forces acting on you, they must be equal in magnitude and opposite in direction. Here, each arrow was drawn with the same length because we expect the forces to be equal in magnitude.

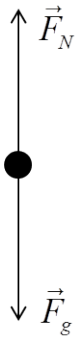


Figure 5.1

Of course, there are more complicated scenarios in which many forces may be acting on an object at once, and those forces may point in several directions. In these cases, it is helpful to use a coordinate system, break forces into components, and consider the “tug-of-war” in each component direction separately.

Figure 5.2 shows a force vector, \vec{F} , in a coordinate system such that it makes an angle, θ , with the x -axis. That vector can be described as the sum of two parts – an x -component (F_x , blue) and a y -component (F_y , red). Given that we are using a Cartesian coordinate system, the x and y components will always be perpendicular to one another.

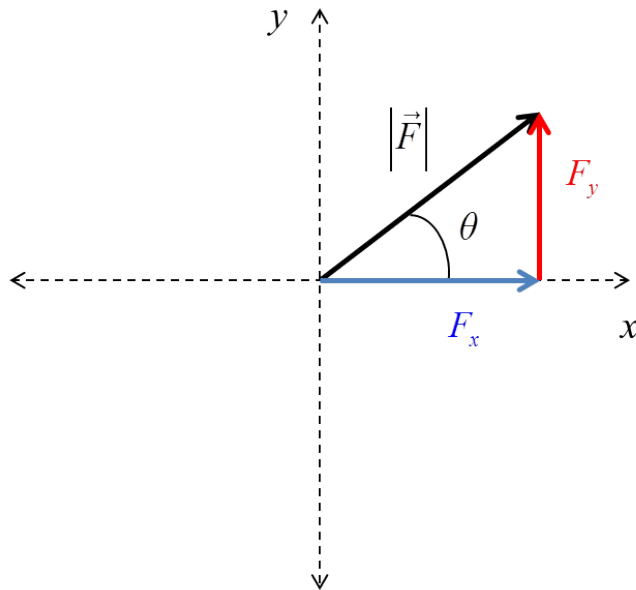


Figure 5.2

The trigonometry for a right triangle provides us with a set of formulae that define the relationships amongst the quantities in the figure, where the angle θ is always measured counterclockwise from the positive x -axis (from 0° to 360°). That is,

$$|\vec{F}| = \sqrt{F_x^2 + F_y^2} \quad (\text{Eq. 1})$$

$$\sin \theta = \frac{F_y}{|\vec{F}|} \quad (\text{Eq. 2})$$

$$\cos \theta = \frac{F_x}{|\vec{F}|} \quad (\text{Eq. 3})$$

where $|\vec{F}|$ represents the magnitude, or the amount, of the force. Using Eq. 1, 2, or 3, we are able to find the components of the force in our coordinate system if we know the magnitude of the force and the angle, or direction, along which the force points. Note that if the force were pointing along a direction in the second quadrant, so that the angle θ is between 90° and 180° , we would find that the x -component of the force is negative, using Eq. 3. If you are unsure, try using a force with a magnitude of 10.0 N at an angle of 135° . Using Eq. 3, you should find that $F_x = -7.07$ N. The negative sign indicates that this force is pointing to the left in the coordinate system. The y -component of this vector would be positive, indicating that it points upward in our coordinate system. With the ability to express a vector in terms of its components, we should consider how this can lead us to the net force on an object (when there is more than one force), and how the components of the net force may be expressed.

The net force on an object is found by performing a *vector sum*, or adding up all of the forces acting on the object. We cannot just simply add the magnitudes of all the vectors. We must account for the fact that the forces act in different directions in order to correctly express the result of the “tug-of-war” on the object. Symbolically, the net force on an object subject to N forces would be

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \cdots + \vec{F}_N \quad (\text{Eq. 4})$$

One of the advantages of finding the components of vectors, with their appropriate signs, is that we can examine the results of the “tug-of-war” in the x and y directions separately. If you think about it, forces pulling only along the y -axis do not have x -components and, thus, cannot affect motion along the x -axis. The reverse is true for forces pulling only along the x -axis. They would not affect motion along the y -axis. Thus, the “tugs-of-war” along the x and y axes may be

considered separately, resulting in the possibility of net force component along either, neither, or both, directions. Expressed as an equation, we are saying

$$F_{net_x} = F_{1_x} + F_{2_x} + \cdots + F_{N_x} \quad (\text{Eq. 5})$$

$$F_{net_y} = F_{1_y} + F_{2_y} + \cdots + F_{N_y} \quad (\text{Eq. 6})$$

where x -components that point to the left and y -components that points downwards in the coordinate system would be negative.

When an object is in *equilibrium*, the net force on the object is 0. Thus, both components of the net force must also be 0. This means that for an object in equilibrium,

$$0 = F_{1_x} + F_{2_x} + \cdots + F_{N_x} \quad (\text{Eq. 7})$$

$$0 = F_{1_y} + F_{2_y} + \cdots + F_{N_y} \quad (\text{Eq. 8})$$

When we know all but one of the x -components of the forces acting on an object in equilibrium, we can use Eq. 7 to solve for the x -component of the unknown force. Likewise, when we know all but one of the y -components of the forces acting on an object in equilibrium, we can use Eq. 8 to solve for the unknown y -component.

Today, you will measure several forces acting on an object, and determine both experimentally and via calculation the necessary force to cause the object to be in equilibrium. You will then compare your calculated and experimental results.

- Goals:**
- (1) Gain further practice in breaking vectors into components in a coordinate system and using the components of a vector to find the magnitude of the vector and the angle describing its direction.
 - (2) Develop a better understanding of equilibrium and how to use the equilibrium conditions to solve for an unknown force.

Procedure

Equipment – force table, balance, masses, mass holders, strings with an attached ring, pulleys

The force table has a circular top with a marked circumference for measuring angular position in degrees. The ring will have strings attached which may be extended over pulleys on the outer

edge of the table. The system is in equilibrium when the ring is centered over the post in the center of the table and is not touching the post, as shown in the figure below.



- 1) Place one of the pulleys at the 180° position and run a string over it so that a mass holder can be attached and hang over the side of the force table. **Record** this angular position and label it as θ_{A1} .
- 2) Choose a mass holder and place 150 g of mass on it. **Measure and record** the mass of the mass holder with its 150 g of mass. Label this as m_{A1} . Hang the mass from the string on the pulley in the previous step.
- 3) Place one of the pulleys at the 90° position and run a string over it so that a mass holder can be attached and hang over the side of the force table. **Record** this angular position and label it as θ_{B1} .
- 4) Choose a mass holder and place 75 g of mass on it. **Measure and record** the mass of the mass holder with its 75 g of mass. Label this as m_{B1} . Hang the mass from the string on the pulley in the previous step.
- 5) Select another mass holder and attach it to a string draped over a third pulley. By trial and error, find both the necessary amount of mass to add to the mass holder and the angular position of the pulley that brings the system into equilibrium. **Record** the angular position of this pulley and use a balance to **measure** the mass of the mass holder you used and the extra masses that you added. Label this experimental angular position as θ_{e1} and the mass as m_{e1} . The “e1” subscripts remind you these are the experimental results of our first set of masses and angles.
- 6) Remove the masses and mass holders to prepare to create a new set of data.

- 7) Place one of the pulleys at the 0° position and run a string over it so that a mass holder can be attached and hang over the side of the force table. **Record** this angular position and label it as θ_{A2} .
- 8) Choose a mass holder and place 70 g of mass on it. **Measure and record** the mass of the mass holder with its 70 g of mass. Label this as m_{A2} . Hang the mass from the string on the pulley in the previous step.
- 9) Place one of the pulleys at the 200° position and run a string over it so that a mass holder can be attached and hang over the side of the force table. **Record** this angular position and label it as θ_{B2} .
- 10) Choose a mass holder and place 150 g of mass on it. **Measure and record** the mass of the mass holder with its 150 g of mass. Label this as m_{B2} . Hang the mass from the string on the pulley in the previous step.
- 11) Place one of the pulleys at the 60° position and run a string over it so that a mass holder can be attached and hang over the side of the force table. **Record** this angular position and label it as θ_{C2} .
- 12) Choose a mass holder and place 200 g of mass on it. **Measure and record** the mass of the mass holder with its 200 g of mass. Label this as m_{C2} . Hang the mass from the string on the pulley in the previous step.
- 13) Select another mass holder and attach it to a string draped over a fourth pulley. By trial and error, find both the necessary amount of mass to add to the mass holder and the angular position of the pulley that brings the system into equilibrium. **Record** the angular position of this pulley and use a balance to **measure** the mass of the mass holder you used AND the extra masses that you added. Label this experimental angular position as θ_{e2} and the mass as m_{e2} .

As always, be sure to organize your data records for presentation in your lab report, using tables and labels where appropriate.

Data Analysis

We will first work with the data from the first equilibrium case – the data with a subscript “1”. If your masses were recorded in grams be sure to convert each of your masses to the SI unit of kg for calculations.

The tension in each of the strings pulling on the ring will be equal in magnitude to the gravitational force on the mass that is attached to a string. Use each of your masses to find the gravitational force on each mass, and thus the tension in each string. This is accomplished by using the formula for the gravitational force on an object on Earth, $F_g = mg = m(9.8 \text{ N/kg})$. Retain the subscripts from your masses when labeling your calculated forces. For example, the force on the string due to the mass m_{A1} should be labeled as F_{A1} .

Question 1: Draw a free-body diagram for one of the mass holders hanging from a string and explain why the tension in the string must be equal to the gravitational force on the mass.

You should now have three forces for the first equilibrium dataset: F_{A1} , F_{B1} , and F_{e1} . Each force is associated with a direction that you measured: θ_{A1} , θ_{B1} , θ_{e1} .

Use Eq. 2 and Eq. 3 to find the x and y components of the force F_{A1} , where the x and y directions are in the horizontal plane of the force table, as viewed from above. The 0° direction is along the positive x -axis and the 90° direction is along the positive y -axis. Label these as F_{A1x} and F_{A1y} .

Use Eq. 2 and Eq. 3 to find the x and y components of the force F_{B1} . Label these as F_{B1x} and F_{B1y} .

Recall that the force F_{e1} was found by trial and error and was the equilibrating force necessary to balance the system. Using the components of F_{A1} and F_{B1} , Eq. 7, and Eq. 8, find the predicted x and y components of the equilibrating force. Label these as F_{p1x} and F_{p1y} . The “p1” subscript reminds us this is the predicted results for our first combination of masses and angles.

Question 2: To find the predicted equilibrating force, why would we use Eq. 7 and Eq. 8 versus using Eq. 5 and Eq. 6? Why set the sum of the force components equal to 0 versus equating the sum to a net force on the object?

Use Eq. 1, 2, and/or 3 to find the magnitude of the predicted equilibrating force (Labeled F_{p1}) and its direction (Labeled θ_{p1}). Later, we will compare this predicted equilibrating force to that found experimentally, F_{e1} and θ_{e1} .

We will now analyze the second equilibrium case that we investigated – the data with a subscript “2”. Begin by converting each of your masses to kg.

Again, use each of your masses to calculate the gravitational force on each mass, and thus, the tension in each string. As before, use the relationship $F_g = mg = m(9.8 \text{ N/kg})$ and retain the subscripts from your masses when labeling your calculated forces. For example, the force on the string due to the mass m_{A2} should be labeled as F_{A2} .

You should now have four forces for the second equilibrium dataset: F_{A2} , F_{B2} , F_{C2} , and F_{e2} . Each force is associated with a direction that you measured: θ_{A2} , θ_{B2} , θ_{C2} , θ_{e2} .

Use Eq. 2 and Eq. 3 to find the x and y components of the force F_{A2} . Label these as F_{A2x} and F_{A2y} .

Use Eq. 2 and Eq. 3 to find the x and y components of the force F_{B2} . Label these as F_{B2x} and F_{B2y} .

Use Eq. 2 and Eq. 3 to find the x and y components of the force F_{C2} . Label these as F_{C2x} and F_{C2y} .

Using the components of F_{A2} , F_{B2} , F_{C2} , Eq. 7, and Eq. 8, find the predicted x and y components of the equilibrating force. Label these as F_{p2x} and F_{p2y} .

Use Eq. 1, 2, and/or 3 to find the magnitude of the predicted equilibrating force (Labeled F_{p2}) and its direction (Labeled θ_{p2}). Later, we will compare this predicted equilibrating force to that found experimentally, F_{e2} and θ_{e2} .

Error Analysis

Calculating percent error is one way to compare the measured value of a quantity to its predicted or theoretical value. When the measured quantity is nearly the same as the theoretical value, the percent error is small and it can be said that the results of the experiment closely matched the prediction. However, when the percent error is large, it can be said that the experimental value does not closely match the theoretical value.

To find the percent error between an experimental and predicted value, take the difference between them and then divide by the predicted value. Multiplying by 100% then expresses the result as a percent, as shown in the equation below.

$$\%err = \left| \frac{x_{experimental} - x_{predict}}{x_{predict}} \right| \times 100\%$$

While it does depend on the experiment being performed, in most cases a percent error of 10% or less is considered to be acceptable and suggests that the experiment verified the predicted results. Percent errors greater than 10% typically suggest that either 1) there is an error in the calculations deriving the predicted value or interpreting the experimental data, or 2) there is

some aspect of the experimental process that is flawed or was not performed correctly, leading to invalid data.

For the first equilibrium scenario, calculate the percent error between the predicted magnitude of the equilibrating force, F_{p1} , and that measured experimentally, F_{e1} .

For the first equilibrium scenario, calculate the percent difference between the predicted direction of the equilibrating force, θ_{p1} , and that measured experimentally, θ_{e1} .

For the second equilibrium scenario, calculate the percent error between the predicted magnitude of the equilibrating force, F_{p2} , and that measured experimentally, F_{e2} .

For the second equilibrium scenario, calculate the percent difference between the predicted direction of the equilibrating force, θ_{p2} , and that measured experimentally, θ_{e2} .

Question 3: Evaluate your percent differences for the first and second scenarios. How well do you feel the experimental results matched the predictions? What aspects of the procedure, or measurement could have caused the difference? Explain.

Questions and Conclusions

Be sure to address Questions 1-3 and describe what has been verified and tested by this experiment. What are the likely sources of error? Where might the physics principles investigated in this lab manifest in everyday life, or in a job setting?

Pre-Lab Questions

Please read through all the instructions for this experiment to acquaint yourself with the experimental setup and procedures, and develop any questions you may want to discuss with your lab partner or TA before you begin the lab. Then answer the following questions and type your answers into the Canvas quiz tool for Forces, Vectors, and Equilibrium, and submit it before the start of your lab section on the day this experiment is to be run.

Consider the following forces applied in the x - y plane, where an angle of $\theta = 0^\circ$ is along the positive x -axis and an angle of $\theta = 90^\circ$ is along the positive y -axis.

PL-1) A force of 5.0 N is applied at an angle of $\theta = 25^\circ$. The x -component of this force is

- (A) 5.0 N,
- (B) 4.5 N,
- (C) 2.5 N,
- (D) 2.1 N,
- (E) 0.0 N,
- (F) -2.1 N,
- (G) -2.5 N,
- (H) -4.5 N,
- (I) -5.0 N.

PL-2) Two forces, **A** and **B**, have magnitudes of 10.0 N and 20.0 N, respectively. What is the **magnitude** of their sum when they both lie along the x -axis, with **A** directed along $\theta = 0^\circ$ and **B** directed along $\theta = 180^\circ$?

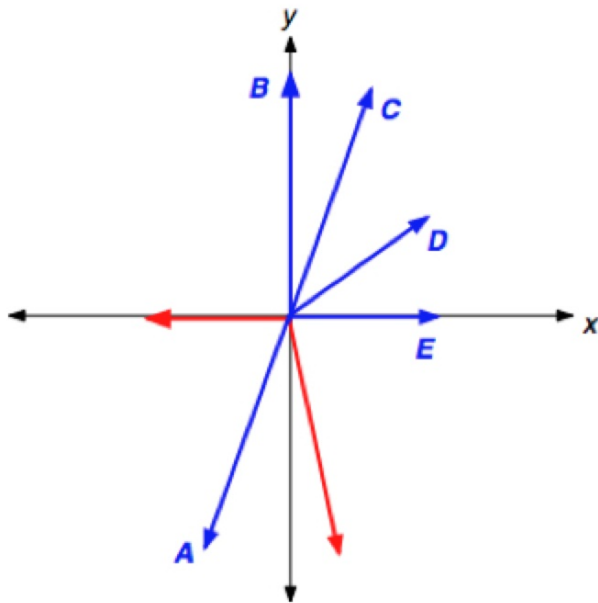
- (A) 30.0 N,
- (B) 22.4 N,
- (C) 20.0 N,
- (D) 10.0 N,
- (E) 0 N,
- (F) -10.0 N,
- (G) -20.0 N,
- (H) -22.4 N,
- (I) -30.0 N.

PL-3) Forces A and B have magnitudes of 10.0 N and 20.0 N, respectively. What is the **magnitude** of their sum when A is directed along $\theta = 0^\circ$ and B is directed along $\theta = 270^\circ$?

- (A) 30.0 N,
- (B) 22.4 N,
- (C) 20.0 N,
- (D) 10.0 N,
- (E) 0 N,
- (F) -10.0 N,
- (G) -20.0 N,
- (H) -22.4 N,
- (I) -30.0 N.

PL-4) Forces A and B have magnitudes of 10.0 N and 20.0 N, respectively. What is the **magnitude** of their sum when A is directed along $\theta = 0^\circ$ and B is directed along $\theta = 50^\circ$?

PL-5) Thelma and Louise do a variation of the "Forces, Vectors and Equilibrium" experiment and set up two force vectors as shown by the **red arrows** in the following figure. Which of the **blue arrows** best represents the force they will have to apply to balance the two red force vectors and bring the system of three forces into equilibrium?



- (A) Vector *A*,
- (B) Vector *B*,
- (C) Vector *C*,
- (D) Vector *D*,
- (E) Vector *E*.